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A study of biped balance control using proportional feedback of ground reaction forces

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Abstract: The ground reaction force is informative for maintaining a biped balance with respect to unknown external forces. From this point of view, we proposed a new control method for static balancing that containing the integral feedback of the ground reaction forces in our previous papers. As the result, the adaptive posture changes are achieved for constant external forces. However, the convergence to the equilibrium posture is not so good because of the integral property of this control method. To improve this problem, the direct, i.e., proportional feedback of ground reaction forces is introduced in this paper. Using the linearization and the transformation to the discrete system, the stability as well as the speed of the convergence is evaluated. The analysis using the root locus method is effectively used to obtain the range of proportional feedback gain of the ground reaction forces for the given parameters of a model. Finally, numerical analyses reveal that the proportional feedback of ground reaction forces slightly improve the speed of the convergence. In conclusion, it is difficult to say that the drastic improvement of the convergence is expected by the proportional feedback of ground reaction forces.

Keywords: Static balance control, Ground reaction force, Proportional feedback, Stability, Speed of convergence

1. INTRODUCTION

The balance maintenance is one of the most important task for the biped control. To design the motions of biped system, the zero moment point (ZMP) [1] criterion is proposed. However, if some disturbing forces are exerted, the reference motion designed according to the ZMP criterion sometimes have to be modified to avoid the tumbling. Thus, some modification rules for reference trajectories were proposed, and these effective results were experimentally investigated using robots (e.g. [2]). However, the mathematical evidences of this method were not shown clearly.

Although for static balance, we proposed a control method containing a feedback, exactly a integral feedback, of ground reaction forces, and proved local stability of the equilibrium posture that emerges by not the modification of the reference posture but essentially the CoP (Center of Pressure) feedback control [3]. In that study, we modeled a biped standing using a body and a foot segment, and assumed the foot segment contacted to the ground with the both ends, as shown in the left side of Fig. 1 (see also the next section in detail). Utilizing the information on the vertical component of the ground reaction forces at these contact points (F_T and F_H) and the joint angle of the ankle (θ), the ankle joint torque τ was determined so that F_T and F_H should become equal with keeping the body segment from tumbling:

$$\tau = -K_d\dot{\theta} - K_p\theta + K_f\int(F_H - F_T)dt. \quad (1)$$

As a result, the posture varied with a constant unknown external force so that the moment around the angle joint became zero, in other words, the moment of the gravity cancelled that of the external force, as shown in the right side of Fig. 1. At the stationary posture, the CoP of the ground reaction forces was controlled to the midpoint of the foot segment.

The characteristics of this control essentially originate from the integral feedback of the ground reaction force. However, due to this integral property, the response of the behavior to the external forces was sometimes delayed. In order to improve this problem, some modification is required for this control law. From this point of view, we here introduce the proportional feedback of the information on the ground reaction forces, and discuss this effect through the mathematical as well as numerical analyses.

2. MATHEMATICAL ANALYSIS

2.1 Problem setting

The same model of the balance control as the previous paper [3] is used here in order to compare the results. It is modeled as an inverted pendulum with a supporting foot segment, which are connected at the ankle joint as shown in Fig. 1(left). The body segment moves only within the sagittal plane. The ankle joint angle θ and its velocity $\dot{\theta}$ are detectable. They are used to construct τ, torque at the ankle joint. The foot segment contacts the ground only at two points (heel and toe), where the vertical components of the ground reaction force of there (F_H and F_T) are also detectable. The foot segment does not slip on the ground and its shape is symmetrical in the anterior-posterior direction. The ankle joint is located in the middle of the foot segment with zero height.

The motion equation of this model is described as

$$\begin{aligned} I\ddot{\theta} &= MLg\sin\theta + F_xL\cos\theta - F_yL\sin\theta + \tau, \\ &= AL\sin(\theta - \theta_f) + \tau \end{aligned} \quad (2)$$

where M is mass of the body segment, L is the distance

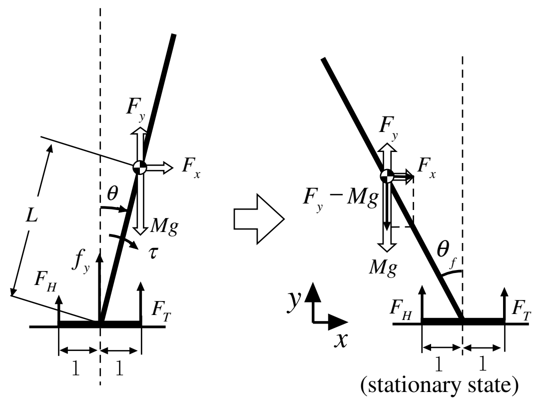

Fig. 1 Model and stationary posture by (1).

from ankle joint to the CoM of the body, g is a gravity constant, F_x and F_y are constants representing unknown external force, A and θ_f are constants defined as

$$A = \sqrt{(Mg - F_y)^2 + F_x^2} \tag{3}$$

$$\tan\theta_f = -\frac{F_x}{Mg - F_y} \tag{4}$$

In addition, the relation between ground reaction forces and ankle joint torque is given as

$$F_H - F_T = \frac{1}{\ell}\tau. \tag{5}$$

In the previous paper [3], we carried on the analysis with substituting (2) and (5) by (1). The stationary state was calculated by putting the derivative with respect to the time to zero, resulting that the posture illustrated in Fig. 1(right) is equilibrium point under the constant F_x and F_y. Furthermore, The asymptotic stability of this steady state is locally ensured for appropriate feedback gains K_d, K_p and K_f.

2.2 A modified control law

The control law (1) achieves a static balance control that can cope with the constant unknown external forces. Although the posture can adaptively change with the external forces, the high responsive behavior is not expected because this adaptive behavior mainly results from the effect of the integral feedback of the ground reaction forces. The improvement of the speed of convergence will be required to apply this control law to the balancing task under fast-varying environments.

One of the possible solutions to overcome this problem may be to introduce the proportional (not integral) feedback of the ground reaction forces: the essence of the control law exists in the integral feedback of ground reaction forces. So, the changing rate of this information, i.e., the proportional feedback of the ground reaction forces is expected to improve the motion speed. From this point of view, we here proposed a modified control law:

$$\tau = -K_d\dot{\theta} - K_p\theta + K_{f1}(F_H - F_T) + K_{f2}\int (F_H - F_T)dt. \tag{6}$$

In the rest of this section, we analyze this control law from the aspect on stability as well as the speed of the convergence.

2.3 A discrete control law

In the analysis for the control law (1), τ in (5) was substituted by this equation. However, such a direct method is not available for the control law (6): the term $F_H - F_T$ in (6) is a value at the moment that the output torque is determined, while the one in (5) is a value after the output torque has been exerted. Thus τ in (5) can not be substituted by (6) due to the difference of the moment at which these forces act, i.e., this equation never holds at every instance because of the time difference.

This problem can be avoided by introducing the discrete-time system which can be distinguish the value of $F_H - F_T$ between the moment the the output torque is determined and the one after the output torque is exerted. Namely, a new variable Π is defined as

$$\Pi = F_H - F_T. \tag{7}$$

Then, (5) becomes

$$\Pi = \frac{1}{\ell}\tau. \tag{8}$$

This equation is expressed in the discrete system as

$$\Pi(k+1) = \frac{1}{\ell}\tau(k). \tag{9}$$

Here, $\tau(k)$ is a discrete representation of the control law (6) that is given as:

$$\tau(k) = -K_d\dot{\theta}(k) - K_p\theta(k) + K_{f1}\Pi(k) + K_{f2}P(k) \tag{10}$$

$$P(k+1) = P(k) + T\cdot\Pi(k) \tag{11}$$

where the time t in the continuous time system is replaced by kT, k is an integer and $T(\neq\ 0)$ is a period of the control.

2.4 A equilibrium point

First of all, the equilibrium point of the discrete control law (10) is calculated. In the the equilibrium point, $\ddot{\theta}$ and $\dot{\theta}$ become zero. Putting the stationary value of θ, Π, P and τ respectively to $\bar{\theta}$, $\bar{\Pi}$, $\bar{P}$ and $\bar{\tau}$, we obtains the following equations:

$$A\sin(\bar{\theta} - \theta_f) + \bar{\tau} = 0 \tag{12}$$

$$\bar{\Pi} = \frac{1}{\ell}\bar{\tau} \tag{13}$$

$$\bar{\tau} = -K_p\bar{\theta} + K_{f1}\bar{\Pi} + K_{f2}\bar{P} \tag{14}$$

$$\bar{P} = \bar{P} + T\cdot\bar{\Pi} \tag{15}$$

Solving the above equations, the equilibrium point is calculated as

$$(\bar{\theta}, \bar{\Pi}, \bar{P}) = (\theta_f, 0, K_p\theta_f/K_{f2}) \tag{16}$$

and then $\bar{\tau} = 0$, i.e., the control input $\tau(k)$ becomes zero. These stationary states are the same as those for the control law (1)

2.5 A discrete form of motion equation

The control law (10) is described as a discrete-time system, while the motion equation is as a continuous-time system. Such a mixed system of the continuous and discrete-time system makes stability analysis difficult. Thus, we transform the motion equation (2) to the discrete-time system.

To begin with, the equation (2) is linearized around the equilibrium point (16). Denoting the deviation from the equilibrium point (16) by using Δ, (2) turns to

$$I\Delta\ddot{\theta} = A\sin(\bar{\theta} + \Delta\theta - \theta_f) + (\bar{\tau} + \Delta\tau) \tag{17}$$

Using the stationary values, this equation becomes

$$I\Delta\ddot{\theta} = A\sin\Delta\theta + \Delta\tau \tag{18}$$

Linearizing this equation, we obtain

$$\ddot{x} = ax + bu \tag{19}$$

where $x = \Delta\theta$, $a = A/I$, $b = 1/I$ and $u = \Delta\tau$. Here, we assume that the control input u is constant during one control interval T, i.e., $u = u_k$ $(kT < t < (k+1)T)$. Then, the solution of this differential equation is given as

$$x = C_1\exp(\lambda t) + C_2\exp(-\lambda t) - b/a\cdot u_k \tag{20}$$

where $\lambda = \sqrt{a}$, C_1 and C_2 are the parameter that is determined the state variables at the time $t = kT$. Then, putting $x_1(k) = x(kT)$ and $x_2(k) = \dot{x}(kT)$, i.e.,

$$x_1(k) = C_1\exp(\lambda kT) + C_2\exp(-\lambda kT) - b/a\cdot u_k \tag{21}$$

$$x_2(k) = \lambda C_1\exp(\lambda kT) - \lambda C_2\exp(-\lambda kT) \tag{22}$$

the variables $x_1(k+1)$ and $x_2(k+1)$ are described using $x_1(k)$ and $x_2(k)$ by eliminating the parameters C_1 and C_2 as follows:

$$x_1(k+1) = cx_1(k) + s/\lambda x_2(k) + (c-1)u(k) \tag{23}$$

$$x_2(k+1) = \lambda s x_1(k) + cx_2(k) + \lambda s\cdot u(k) \tag{24}$$

where $x_1(k) = \theta(k)$, $x_2(k) = \dot{\theta}(k)$, $c = \cosh(\lambda T)(\neq 1)$, $s = \sinh(\lambda T)(\neq 0)$, $u(k) = 1/A\cdot\tau(k)$,

2.6 Stability analysis

As the result of the previous section, we obtain the difference equation:

$$X(k+1) = GX(k) + Hu(k). \tag{25}$$

where

$$X(k) = \begin{bmatrix} x_1(k) & x_2(k) & \Pi(k) & P(k) \end{bmatrix}^T, \tag{26}$$

$$G = \begin{bmatrix} c & s/\lambda & 0 & 0 \\ \lambda s & c & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & T & A_l \end{bmatrix} \tag{27}$$

$$H = \begin{bmatrix} c-1 & \lambda s & A_l & 0 \end{bmatrix}^T \tag{28}$$

$$u(k) = -K_d'x_2(k) - K_p'x_1(k) + K_{f1}'\Pi(k) + K_{f2}'P(k) \tag{29}$$

Here, $A_l = A/\ell$ and K_d/A, K_p/A, K_{f1}/A, K_{f2}/A were replaced by K_d', K_p', K_{f1}', K_{f2}'.

The controllability matrix of this linear discrete system is

$$C = \begin{bmatrix} c-1 & c_{12} & c_{13} & c_{14} \\ \lambda s & c_{22} & c_{23} & c_{24} \\ A_l & 0 & 0 & 0 \\ 0 & TA_l & TA_l & TA_l \end{bmatrix} \tag{30}$$

$$c_{12} = (c-1)(2c+1) \tag{31}$$

$$c_{13} = (c-1)(4c^2 + 2c - 1) \tag{32}$$

$$c_{14} = (c-1)(8c^3 + 4c^2 - 4c - 1) \tag{33}$$

$$c_{22} = \lambda s(2c-1) \tag{34}$$

$$c_{23} = \lambda s(4c^2 - 2c - 1) \tag{35}$$

$$c_{24} = \lambda s(8c^3 - 4c^2 - 4c + 1) \tag{36}$$

The determinant of the controllability matrix becomes

$$det(C) = 4\lambda s A_l^2 T(c-1)^2 \tag{37}$$

Because the control period T is not equal zero, $s \neq 0$, $c \neq 1$ and so $det(C) \neq 0$. This indicates that the equilibrium point can be stabilized by selecting suitable K_d, K_p, K_{f1} and K_{f2}.

2.7 Speed of the convergence

Substituting the input of the discrete-time system (25) by (29), the next equation is obtained:

$$X(k+1) = G'X(k). \tag{38}$$

where

$$G' = \begin{bmatrix} g_{11}' & g_{12}' & g_{13}' & g_{14}' \\ \lambda s(1-K_p') & c - K_d'\lambda s & K_{f1}'\lambda s & K_{f2}'\lambda s \\ -K_p'A_\ell & -K_d'A_\ell & K_{f1}'A_\ell & K_{f2}'A_\ell \\ 0 & 0 & T & 1 \end{bmatrix} \tag{39}$$

$$g_{11}' = c - K_p'(c-1) \tag{40}$$

$$g_{12}' = s/\lambda - K_d'(c-1) \tag{41}$$

$$g_{13}' = K_{f1}'(c-1) \tag{42}$$

$$g_{14}' = K_{f2}'(c-1) \tag{43}$$

The characteristic equation of this system is given as

$$|zI - G'| = 0 \tag{44}$$

The speed of the convergence is evaluated by the roots of this characteristic equations, especially, the dominant poles of the transfer function. However, the order of this equation is four, which is too high to calculate them analytically. This is a reason why we introduce the numerical method to evaluate the speed of the convergence in the next section.

3. NUMERICAL ANALYSIS

The previous section reveals that the control law (10) stabilizes the same equilibrium posture as the control law (1). However, the motivation of introducing the positional feedback of the ground reaction forces was to improve the speed of convergence to the equilibrium posture. The mathematical analysis in the previous section did not provides any informative answers on it because of the complexity of the difference equation.

In this section, we adopt a numerical analysis to overcome this problem.

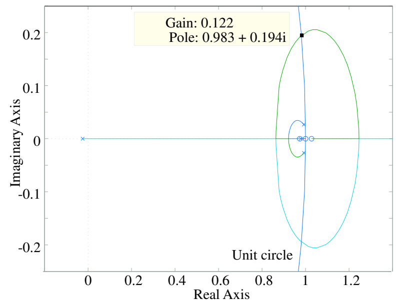

Fig. 2 Root locus of the discrete system (25).

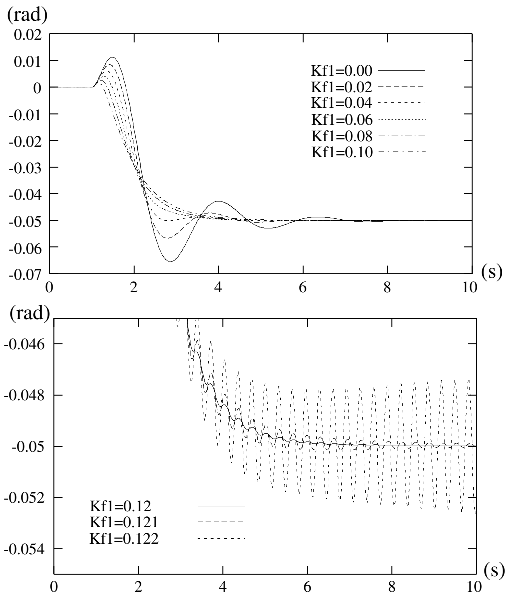

Fig. 3 Time response for different K_{f1}.

3.1 Methods

In oder to execute the numerical analysis, realistic values are set to every parameters. For the link parameters, the following values are set: $M = 60$ [kg], $L = 1$[m], $I = \frac{4}{3}ML^2$ [kg $\cdot$ m^2].

The purpose of this section is to assess the effect of the proportional feedback of the ground reaction forces introduced newly in this paper. Therefore, we firstly chose the feedback gains except the proportional feedback of ground reaction forces by many trials as follows: $K_d = 500$, $K_p = 1500$, $K_{f2} = 0.3$. Then, we increase it, i.e., K_{f1} from 0 where the proportional feedback of ground reaction forces gives no effects.

The external forces are initially set $F_x = 0, F_y = 0$. In order to evaluate the response with respect to the change of external force, F_x is abruptly changed to $0.05Mg$ [N] when 1 [s] passed from the start of the computer simulation. The differential equation (2) is numerically solved by the 4th-order Runge-Kutta method with the step size 0.001[s], while the control input is determined every 0.01[s], i.e., $T = 0.01$. The time evaluation of the ankle joint angles θ is computed for some K_{f1} values to evaluate the effect of the direct feedback of the ground reaction forces.

3.2 Results

For a somewhat large K_{f1}, the balance of the body segment was disturbed. Thus, we first studied the maximum value of the feedback gain K_{f1} that can keep the body segment stable. To examine the changes of the roots of the characteristic equation (44), the root locus method was adopted. The root locus that was drawn by MATLAB is shown in Fig. 2. Note that the system become unstable for the roots outsider the unit circle, since it is discrete-time system. This analysis reveals that the stability is not ensured for the K_{f1} larger than about 0.122.

Using the values from 0 to 0.10, the time evaluation with respect to the abrupt change of the external force was simulated. The time course of the ankle joint angle θ is illustrated at the top of Fig. 3. As shown in this figure, the settling time is surely improved for large K_{f1}. However, it is about 3 (s) for large K_{f1} values, implying that the drastic improvement is not expected by the effect of the proportional feedback of the ground reaction forces introduced here.

The simulation result for the K_{f1} from 0.120 to 0.122 is illustrated at the bottom of Fig. 3. The trajectory become oscillatory for $K_{f1} = 0.121$. More over, the amplitude of the oscillation gradually increase at $K_{f1} = 0.122$, implying that the time response is instabilized for K_{f1} larger than 0.122. These results are consistent with the analysis of the root locus

4. CONCLUSION

In this paper, we investigated a new balance control law containing the proportional feedback of the ground reaction forces. The posture at which the moment of the gravity and the external forces are balanced becomes an equilibrium point of this control law. Using the linearization and the transformation to the discrete-time system, the stability as well as the speed of the convergence was evaluated. The analysis using the root locus method provided the stabilizing range of the proportional feedback gain of ground reaction forces. Numerical analyses revealed that the proportional feedback of ground reaction forces slightly improve the speed of the convergence. Consequently, it is difficult to say that the drastic improvement of the convergence is expected by the proportional feedback of ground reaction forces.

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