

# Where Center of Pressure Should Be Controlled in Biped Upright Posture

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## SUMMARY

The stability of the upright posture of the biped system on horizontal ground has been evaluated in terms of the vertical projection of the center of gravity on the ground. Since the point of projection coincides with the center of pressure of the ground reaction force (CoP), so long as static equilibrium is maintained, robust control can be realized if the ground reaction force is directly controlled. Consequently, we modeled the upright standing state of the biped system by a two-link system composed of the foot and the remainder and proposed a method of controlling the ground reaction force based on the output of the ankle joint. The method is equivalent to control of the CoP, but no detailed study of where the center should be controlled has been presented. This point is crucial if the foot is not symmetrical in the anterior–posterior direction, or if the ankle joint is located at a certain height, as in the case of humans. The purpose of this paper is to investigate this problem. Two control methods can be considered, depending on whether the evaluation criterion is defined as the minimum steady output of the ankle joint or as the stability margin. For each of the criteria, we analyze the stability and the steady-state posture. To clarify the criterion by which humans perform control, the CoP was measured for humans in the upright posture. The results suggest that humans are less likely to use the minimum output of the ankle joint as the evaluation

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**Key words:** upright posture; center of pressure; human measurement; ankle joint output; stability margin.

## 1. Introduction

In the course of growth, humans learned to stand on two feet at first, and then acquired the ability to walk on two feet. In other words, it is essential to the biped system to maintain the upright posture. For biped systems for horizontal ground, the static balance has been discussed on the basis of the positional relation between the vertical projection of the center of gravity of the system and the support polygon [1]. The support polygon is the convex polygon with the minimum area that contains all support points (grounded points).

If the vertical projection of the center of gravity stays within the support polygon, stability is ensured and the system does not fall over. Thus, one method for the biped system to maintain the upright posture is to design the posture, such as the joint angles, so that the projection of the center of gravity remains within the sole of the foot, and to apply positional feedback so that the above situation is maintained.

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If the ground reaction can be measured, important information is obtained for control of the upright posture. When static stability is maintained, the center of pressure of the ground reaction force (CoP) agrees with the vertical projection of the center of gravity [2]. In the case of dynamic stability, the ZMP (zero moment point) [3], whose moment, a combination of the gravitational force and the inertial force, is zero, agrees with the CoP [2]. Consequently, if the CoP can be directly controlled, control will be more robust to external disturbances and modeling error than the above method based on position control.

Based on this idea, we previously proposed a control method based on the ground reaction for maintaining the upright posture of the biped system [4, 5]. In the proposed method, the output of the ankle joint in the steady upright posture is emphasized, and an attempt is made to control the CoP at the intersection of the extension of the resultant of the external force and the gravitational force from the ankle joint with the ground. However, this method is not very desirable from the viewpoint of stability. In this paper, we formulate this problem and discuss how the control evaluation criterion is related to the position of the CoP. In accordance with this discussion, the control criterion for human upright posture is examined on the basis of measurements of that posture.

## 2. Mathematical Background

### 2.1. Control model for CoP

The same model for the upright posture of the biped model as in our previous paper [5] is considered below. The model is simplified by making the following assumptions. First, the part other than the foot is defined as the “upper body” and is represented by a single link, by assuming that the joint angles other than the ankle joint are kept constant. In fact, the center of gravity is shifted greatly by a small movement of the ankle joint, because the ankle joint is located at the lowest position of the body, indicating that the ankle joint is the most effective means of stabilizing of the upright posture. This assumption allows us to focus on the role of the ankle joint in balance control.

Next, it is assumed that toppling can be represented as motion in some certain vertical plane, which is defined as the sagittal plane. Assuming left–right symmetry of the biped system, only half of the system is modeled.

In order to simplify the contact with the ground, two-point contact at the two ends of the foot is assumed. It is assumed that the vertical component of the ground reaction force can be measured at each contact point (denoting the component at the heel as  $F_H$  and that at the toe as  $F_T$ ). In the steady state, the position of CoP is expressed by the difference between  $F_H$  and  $F_T$ . The sole of the foot is not

symmetrical in the anterior–posterior direction, with  $\ell_T$ ,  $\ell_H$ , and  $\ell_G$  being the horizontal distances of the toes, heel, and center of gravity of the foot from the ankle joint. Let  $\ell_A$  be the vertical distance between the ground and the ankle joint. Let the length of the foot be  $2\ell$ . Then,  $2\ell = \ell_H + \ell_T$ . It is assumed that the ground friction is sufficiently large that slippage of the foot never occurs. Figure 1 illustrates the model to be considered.

In order to examine the effectiveness of CoP control, a constant external force is assumed and the responding behavior of the walking system is analyzed. Let the horizontal and vertical components of the constant external force be  $F_x$  and  $F_y$ , respectively. So long as the equilibrium of the walking system is maintained, the foot part maintains a stationary state, and only the upper part moves.

Its motion is represented as follows:

$$I\ddot{\theta} = MLg \sin \theta + F_x L \cos \theta - F_y L \sin \theta + \tau \quad (1)$$

Here,  $M$  is the mass of the upper part,  $I$  is the moment of inertia of the upper part around the ankle joint,  $L$  is the distance from the ankle joint to the center of gravity of the upper part,  $\theta$  is the displacement of the upper part from the vertical direction, that is, the angle of the ankle joint,  $\tau$  is the torque of the ankle joint, and  $g$  is the acceleration of gravity.

By the relation of moment balance at the two ends of the foot, the ground reaction forces  $F_T$  and  $F_H$  can be expressed respectively as follows:

$$F_T = -\frac{1}{2\ell}\tau + m_T g + \frac{\ell_H}{2\ell}f_y - \frac{\ell_A}{2\ell}f_x \quad (2)$$

$$F_H = \frac{1}{2\ell}\tau + m_H g + \frac{\ell_T}{2\ell}f_y + \frac{\ell_A}{2\ell}f_x \quad (3)$$

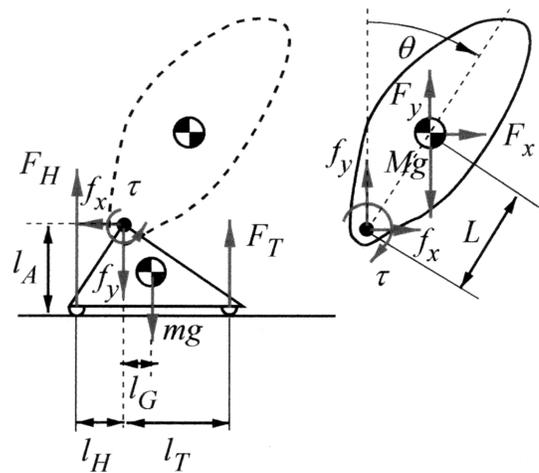


Fig. 1. Biped standing model.



margin in the steady state is increased, and the posture in the steady state is investigated.

When the CoP is located at point C, the difference between  $F_T$  and  $F_H$  is 0. Consequently, the control rule is defined by equating the target value  $F_0$  for the difference in Eq. (11) to zero:

$$\tau = -K_d\dot{\theta} + K_p(\theta_d - \theta) + K_f \int (F_H - F_T) dt \quad (15)$$

This control is called ‘‘control rule 2.’’

Below, in order to simplify the analysis, the variable  $\tau_f$  defined by

$$\tau_f = \int (F_H - F_T) dt \quad (16)$$

is introduced;  $\theta$ ,  $\dot{\theta}$ , and  $\tau_f$  are considered as state variables. Substituting Eqs. (15) and (16) into Eq. (7), we obtain

$$I\ddot{\theta} = AL \sin(\theta - \theta_f) - K_d\dot{\theta} + K_p(\theta_d - \theta) + K_f\tau_f \quad (17)$$

On the other hand, by differentiating both sides of Eq. (16) with respect to time and applying Eqs. (2), (3) and then Eqs. (15) and (16), we obtain

$$\begin{aligned} \dot{\tau}_f = & \frac{1}{\ell}(-K_d\dot{\theta} + K_p(\theta_d - \theta) + K_f\tau_f) \\ & + (m_H - m_T)g + \frac{\ell_T - \ell_H}{2\ell} f_y + \frac{\ell_A}{\ell} f_x \end{aligned} \quad (18)$$

In the steady state, it follows from Eqs. (4) and (5) that  $f_x = -F_x$  and  $f_y = Mg - F_y$ . Consequently, the equilibrium point is determined by solving the following two equations:

$$\begin{aligned} AL \sin(\bar{\theta} - \theta_f) + K_p(\theta_d - \bar{\theta}) + K_f\bar{\tau}_f &= 0 \quad (19) \\ \frac{1}{\ell}(K_p(\theta_d - \bar{\theta}) + K_f\bar{\tau}_f) + (m_H - m_T)g & \\ + \frac{\ell_T - \ell_H}{2\ell}(Mg - F_y) - \frac{\ell_A}{\ell}F_x & \\ = 0 & \quad (20) \end{aligned}$$

By Eqs. (19) and (20), the following equation applies in the steady state:

$$\begin{aligned} AL \sin(\bar{\theta} - \theta_f) & \\ = (m_H - m_T)g\ell + \frac{1}{2}(\ell_T - \ell_H)(Mg - F_y) & \\ - \ell_A F_x & \quad (21) \end{aligned}$$

The left-hand side of the above equation represents the torque which is needed to maintain  $\theta = \bar{\theta}$  for the upper part.

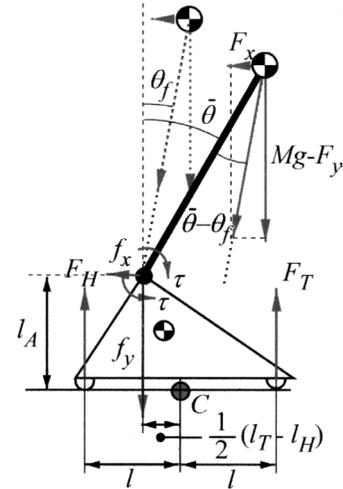


Fig. 3. Stationary state by Control 2.

The right-hand side is the sum of the moments produced by the mass of the foot,  $Mg - F_y$  and  $F_x$  around the central point C of the foot. In other words, the above equation implies that the moment of rotation of the foot around point C is canceled by inclining the upper part.

The torque of the ankle joint which is needed in this posture is given by

$$\begin{aligned} \tau = & -(m_H - m_T)g\ell \\ & - \frac{1}{2}(\ell_T - \ell_H)(Mg - F_y) + \ell_A F_x \end{aligned} \quad (22)$$

Substituting the above expression into Eqs. (2) and (3), we obtain

$$\begin{aligned} F_T = F_H & \\ = \frac{1}{2}(m_H + m_T)g + \frac{1}{2}(Mg - F_y) & \quad (23) \end{aligned}$$

This implies that the CoP is located at the center of the foot, that is, i.e., at point C. Figure 3 shows the posture in the steady state. The stability of the steady state is discussed in the Appendix.

### 3. CoP in Human Upright Posture

#### 3.1. Purpose of measurement

Two control methods were proposed in the previous section. Control rule 1 gives the minimum ankle joint output in the steady state, but makes the CoP deviate in a certain direction due to the foot structure. The position also

depends on the external force. Depending on the situation, there can be a danger of toppling, and the method cannot be considered best in terms of stability. In control rule 2, on the other hand, the maximum stability margin is obtained, since the CoP is kept at the center of the foot. But a nonzero ankle joint torque is required.

Each of these control rules is best in terms of some evaluation criterion, but has both advantages and disadvantages. From the viewpoint of maintaining the upright posture, it is crucial to select the location at which the CoP should be maintained.

Thus, the question is what strategy is used by humans whose foot shape is not symmetrical in the anterior–posterior direction and the ankle joint is located at a certain height. In this study, control rules 1 and 2 are used as hypotheses regarding the control strategy for maintaining human upright posture, and the significance of the hypotheses is examined. The two methods differ in the location of the CoP. Consequently, the position of the CoP in the upright posture was measured.

The CoP is generally measured by investigating the balance function with a body sway meter, and the results are used to evaluate the body sway [7]. In this field, the important items in such measurement are the area, width, and shape of the sway, and the difference between the sway with the eye open and closed. Therefore, the average (stationary) position of CoP is not considered to be a great matter. There are reports of cases in which a periodic external disturbance is applied by moving the floor in the anterior–posterior direction [8], and in which the load is suddenly removed in a pulling task in the upright posture and the recovery of the posture is investigated [9]. However, the change in the relative position of CoP with respect to the foot according to the given external force has not been investigated. In this study, the distance of the CoP from the ankle joint or the center of the foot is investigated by measurement.

### 3.2. Method of measurement

A pressure distribution measurement system (F-scan NITTA) was used in the measurement of the distribution of the ground reaction force. The system contains a foot-shaped sensor sheet. By inserting the sheet between the ground and the foot, the distribution of the ground reaction force could be determined. The surface of the sheet was sectioned into approximately 5-mm squares in the anterior–posterior and the lateral directions. After calibrating the system, the contact force could be measured at 256 levels for each mesh point. For measurement of the upright posture, a three-dimensional position measurement system (OPTOTRAK, Northern Digital Systems) was used. The three-dimensional position was measured with a precision of 1 mm. The purpose of this experiment was to examine

the human upright posture in the steady state. Consequently, the sampling frequency of 10 Hz achievable in both measurement systems was considered sufficient.

The external disturbance applied to the upright posture in the steady state was realized in the simulation by using a slope stand and providing an inclination to the stage. When the inclination angle is  $\alpha$ , the corresponding external disturbance is represented as  $F_x = Mg \sin \alpha$ ,  $F_y = Mg(1 - \cos \alpha)$ . The inclination angle is set by using the angle indicator attached to the slope stand. Since the data analysis is restricted to the sagittal plane, the pressure distribution measurement system was set perpendicular to the measurement axis of the position measurement system. Since only a single sensor sheet was available, it was set at the position of the standing left foot.

Five subjects (males, 22 to 24 years old) participated in the experiment. The subjects were not informed of the purpose of the experiment. During the measurement, each subject was instructed to stand on the slope stand with the bottom of the foot in contact with the stage, and to gaze at a vertical white wall at a distance of 3 m. The subjects were also instructed not to bend the knee joints or hip joints, so that joints other than the ankle joint do not affect the balance control.

Five markers were used for position measurement. Four were attached to the shoulder joint (acromion), the hip joint (greater trochanter), the knee joint (lateral gap), and the ankle joint (lateral malleolus). The other marker was attached to the origin of the pressure distribution measurement system on the slope stand. This marker helped to determine the horizontal distance between the CoP location and the ankle joint.

The experimental procedure was as follows. First, the slope stand was set horizontally ( $0^\circ$ ). The subject stood still on the stand. In the motionless state, the ground reaction force and the posture were measured for 10 seconds. This measurement was defined as a set, and three sets of measurements were performed. In order to reset the creep characteristic of the sensor sheet, the subject was instructed to step down from the stand during the measurement interval.

After the three measurement sets, the slope angle was adjusted. The slope was set so that the toes were lifted. The slope angle was successively increased to  $5^\circ$ ,  $10^\circ$ , and  $15^\circ$ . For each slope angle, three measurement sets lasting 10 seconds each were performed.

When the measurements for the slope angle of  $15^\circ$  were completed, the slope stand was returned to the horizontal state ( $0^\circ$ ) and the three sets of measurements were performed again. In this case, however, the subject was instructed that his weight should be concentrated on the heel. When the measurements were completed, the subject was asked whether the first measurement or the last measurement was easier. Figure 4 shows the experimental environment.

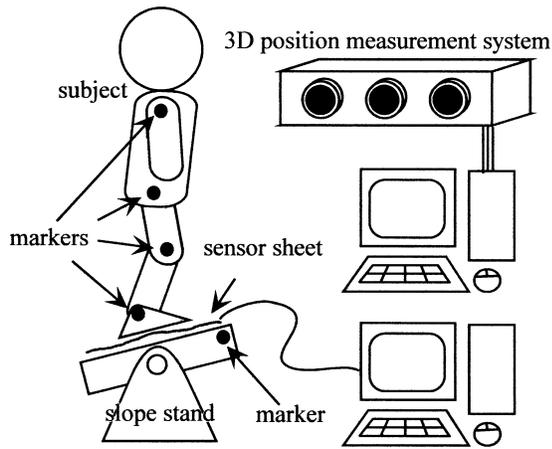


Fig. 4. Experimental environment.

### 3.3. Measurement results

Based on the position data for each joint obtained from the three-dimensional position measurement system, the angles of the hip joint and the knee joint are calculated. We see that the average joint angle stays within  $5^\circ$ , less than the inclination of the stage, for any slope angle and for all subjects. Thus, it is concluded that the posture of the upper part of the body is kept almost constant.

The position of the CoP was calculated from the data obtained by the pressure distribution measurement system. Then, using the data for the marker position attached to the

slope stand, the horizontal distance in the anterior–posterior direction from the ankle joint to the CoP was determined.

Table 1 shows the mean and standard deviation for each set and for all three sets, for each slope angle. The unit is the millimeter, and the anterior direction from the ankle joint is defined as positive. By “heel-weighted” is meant the result when the stage is set horizontally and the weight is concentrated at the heel. By “midpoint” is meant the distance from the ankle joint to the center of the foot, as measured by a ruler.

The data obtained for all three sets were normalized to the distance from the ankle joint to the foot center. Figure 5 shows the results as graphs. Panel (a) plots the dependence on the slope angle, and panel (b) compares the case in which the subject was instructed to concentrate the weight on the heel and the normal case in the horizontal state of the stage.

Although the location of the CoP in the horizontal state and its dependence on the slope angle vary greatly from subject to subject, we can find two main tendencies, in which the CoP shifts backward (subjects 1, 2, and 4) and forward (subjects 3 and 5). Except for some data for subject 5, however, the CoP is generally maintained 50% or more closer to the center of the foot (normalized distance).

We observe from Fig. 5(b) that when the subject is instructed to place the center of gravity above the heel in the horizontal state of the stage, the CoP certainly shifts to a position closer to the ankle joint than the foot center. After the experiment, all subjects reported that it was more tiring to set the center of gravity above the heel.

Table 1. Forward deviation of CoP from ankle joint position (mm)

Conditions	0 deg.	5 deg.	10 deg.	15 deg.	Heel-weighted	Midpoint	
Subject 1	1st	81 ± 1	73 ± 1	48 ± 3	49 ± 1	15 ± 3	77
	2nd	77 ± 1	73 ± 1	65 ± 2	49 ± 2	11 ± 3	
	3rd	71 ± 1	70 ± 2	55 ± 3	45 ± 1	8 ± 2	
	average	76 ± 4	72 ± 2	56 ± 7	47 ± 2	11 ± 4	
Subject 2	1st	71 ± 1	62 ± 1	50 ± 2	58 ± 3	19 ± 2	80
	2nd	70 ± 2	63 ± 4	45 ± 2	54 ± 3	29 ± 1	
	3rd	62 ± 1	60 ± 1	52 ± 1	36 ± 3	26 ± 2	
	average	68 ± 4	62 ± 3	49 ± 3	49 ± 10	24 ± 5	
Subject 3	1st	54 ± 2	83 ± 2	79 ± 2	78 ± 2	42 ± 3	75
	2nd	65 ± 2	68 ± 2	71 ± 2	76 ± 2	49 ± 1	
	3rd	75 ± 1	65 ± 2	83 ± 2	83 ± 2	31 ± 2	
	average	65 ± 9	72 ± 8	78 ± 6	79 ± 4	41 ± 8	
Subject 4	1st	50 ± 2	51 ± 1	42 ± 1	45 ± 1	51 ± 1	69
	2nd	60 ± 1	45 ± 1	49 ± 2	32 ± 2	24 ± 2	
	3rd	59 ± 2	43 ± 2	36 ± 1	42 ± 2	38 ± 1	
	average	56 ± 5	46 ± 4	43 ± 6	40 ± 6	38 ± 11	
Subject 5	1st	18 ± 2	8 ± 2	37 ± 1	51 ± 3	13 ± 3	75
	2nd	34 ± 3	43 ± 4	46 ± 2	52 ± 2	8 ± 2	
	3rd	33 ± 3	49 ± 1	43 ± 1	51 ± 3	1 ± 4	
	average	29 ± 8	33 ± 18	42 ± 4	52 ± 3	8 ± 6	

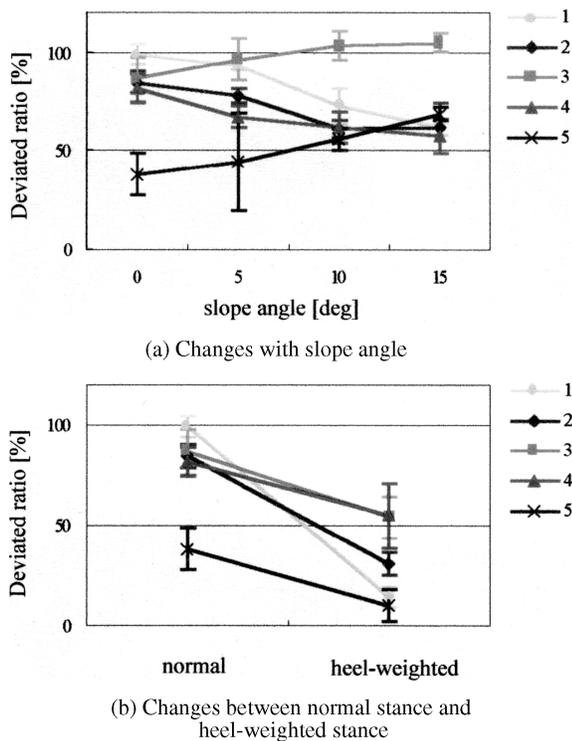


Fig. 5. Normalized distance of CoP from ankle joint.

## 4. Discussion

### 4.1. Control strategy for human upright posture

Assume the case in which control rule 1 is applied. Ignoring the mass of the foot, the CoP should have been kept directly below the ankle joint if the stage was in the horizontal state. According to the experimental results, however, the CoP is located close to the center of the foot (subjects 1 to 4). Furthermore, when the slope was set so that the toes are lifted, the CoP of the ground reaction force should have shifted backward from the ankle joint according to control rule 1. However, the experimental results reveal that the CoP remains in front of the ankle joint, and closer to the foot center than the ankle joint. Even in the case of subject 5, for whom the CoP was located closest to the ankle in the horizontal state, the normalized distance exceeded 50% when the slope angle was 10° and 15°. This result indicates that the upper part is tilted slightly forward so that the center of gravity is shifted toward the toes. Thus, it is unlikely that control rule 1, that is, minimization of the output torque of the ankle joint, is adopted in control strategy of human upright posture.

On the other hand, if control rule 2 was applied, the CoP should have been kept at a constant position regardless of the external force. However, no such tendency is observed in the experimental results. Also in the experiments of Hay and Redon, the CoP was surely moved after no external force is exerted by releasing a load in the hand in the upright posture [10]. These results indicate that humans do not completely use control rule 2. In other words, the CoP is kept in the neighborhood of the foot center, but the location is adjusted slightly according to the situation. It appears that there is an evaluation criterion other than stability.

Furthermore, all subjects reported that it was more tiring to set the center of gravity toward the heels. This implies that control rule 1 is not necessarily the energy minimization criterion in humans. Considering that the human body is a multilink structure composed of a large number of joints, this result is natural because the energy criterion should include evaluations of other joint torques. However, it is to be expected that when the upper part is tilted, the ankle joint at the lowest position will give the largest output. In industrial robots, for example, the most powerful motor is arranged at the joint closest to the base. In that sense, the report of the subjects that the posture can be more easily maintained if the upper part is slightly tilted toward the front is contrary to the above interpretation. It is possible that the human musculoskeletal system is structured so that the upright posture is more easily maintained by keeping the CoP slightly forward.

### 4.2. Application to upright posture control of robots

In the case of walking robots, the foot shape can be designed arbitrarily. The humanoid robots which have been publicized generally have feet that are not symmetrical in the anterior–posterior direction like humans due to the purposes of the investigations [11–13]. Even if the feet have a symmetrical shape in the anterior–posterior direction, the ankle joint may be set at a position higher than the ground due to implementation constraints. The question arises as to where the CoP should be located in such a case. Human measurements include individual differences, and it is not easy to derive a general conclusion, but in most cases the CoP tends to be kept in the neighborhood of the foot center. From the viewpoint of energy efficiency, however, it is not necessarily true that the CoP robot should be kept at the foot center, because nonzero ankle joint torque is required to maintain the upright posture. In order to investigate this point, the upright posture model should be represented by a multilink structure and analyzed. Undoubtedly energy efficiency and stability will be the items to be considered as criteria. An adequate reference position of CoP control

may be defined at the final stage as a trade-off between these criteria. In any case, in the investigation of the upright posture to be maintained there undoubtedly exists a solution that simultaneously optimizes the ankle joint output and stability if the foot shape is designed to be symmetrical in the anterior–posterior direction and the ankle joint is set as low as possible.

## 5. Conclusions

This paper has considered a biped system that maintains an upright posture, with the foot shape not symmetrical in the anterior–posterior direction or with the ankle joint at a relatively high position above the ground. The control method based on the ground reaction force was investigated from the viewpoint of the ankle joint. Two-point grounding has been considered, in which the CoP can be controlled by adjusting the difference of the ground reaction force in the vertical direction between the two points. Depending on the difference setting used, there can be two control rules, namely, a control rule that minimizes the ankle joint output in the steady state, and a control rule that maximizes the stability margin. The stability and the posture in the steady state were investigated.

To determine what evaluation criterion is actually used in upright human posture, the position of the CoP was measured in this posture. The following observations were made. A human tilts the upper body slightly in the forward direction in the upright posture, which indicates that a control method minimizing the torque of the ankle joint is not used. It is also seen that a control method that minimizes the ankle joint torque is not an easy posture for the human body as a multilink system.

The actual upright posture is affected by visual information and vestibular sensation [14]. The control rules considered in this paper do not include such information processing, but may potentially be used as simple upright posture control models. There are two possible future paths of development. One is to refine this simple control model and make it more sophisticated. The other is to demonstrate that the control method presented in this paper actually exists as a component of the human control method. One method of taking the latter approach may be to investigate the dynamic characteristics by comparing the measured stiffness of the ankle joint and the feedback gain of the control rule, as was done by Morasso and Schieppati [15].

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## APPENDIX

### Stability of Equilibrium Point According to Control Rule 2

From the viewpoint of control, it is important to ensure the stability of the equilibrium point. The stability of the equilibrium point given by Eqs. (19) and (20) is analyzed below.

Linearizing Eqs. (17) and (18) near the equilibrium point, we obtain

$$I\ddot{\theta} = (AL \cos(\bar{\theta} - \theta_f) - K_p)\theta - K_d\dot{\theta} + K_f\tau_f \quad (\text{A.1})$$

$$\begin{aligned} \dot{\tau}_f = & \frac{1}{\ell}(-K_d\dot{\theta} - K_p\theta + K_f\tau_f) \\ & - \frac{\ell_T - \ell_H}{2\ell}ML\ddot{\theta} \sin \bar{\theta} + \frac{\ell_A}{\ell}ML\ddot{\theta} \cos \bar{\theta} \end{aligned} \quad (\text{A.2})$$

In the above calculation,  $f_x$  and  $f_y$  are linearized near  $\bar{\theta}$  as follows:

$$\bar{f}_x = ML\ddot{\theta} \cos \bar{\theta} - F_x \quad (\text{A.3})$$

$$\bar{f}_y = -ML\ddot{\theta} \sin \bar{\theta} + Mg - F_y \quad (\text{A.4})$$

The characteristic equation for the above linear differential equation is

$$\lambda^3 + p_2\lambda^2 + p_1\lambda + p_0 = 0 \quad (\text{A.5})$$

$$p_2 = \frac{K_d\ell - K_f(I + f(\bar{\theta}))\ell}{I\ell} \quad (\text{A.6})$$

$$p_1 = \frac{K_p - AL \cos(\bar{\theta} - \theta_f)}{I} \quad (\text{A.7})$$

$$p_0 = \frac{K_f AL \cos(\bar{\theta} - \theta_f)}{I\ell} \quad (\text{A.8})$$

$$f(\bar{\theta}) = \frac{ML}{\ell} \left( \frac{1}{2}(\ell_T - \ell_H) \sin \bar{\theta} - \ell_A \cos \bar{\theta} \right) \quad (\text{A.9})$$

Applying the Routh–Hurwitz stability criterion, the necessary and sufficient condition for the equilibrium point to be (locally) stable is

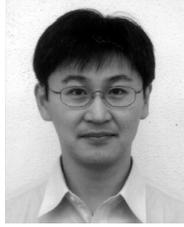
$$K_d > \left( \frac{I}{\ell} + f(\bar{\theta}) \right) K_f \quad (\text{A.10})$$

$$K_p > AL \cos(\bar{\theta} - \theta_f) \quad (\text{A.11})$$

$$K_f > 0 \quad (\text{A.12})$$

$$\begin{aligned} & (K_d - K_f f(\bar{\theta}))(K_p - AL \cos(\bar{\theta} - \theta_f)) \\ & > \frac{I}{\ell} K_p K_f \end{aligned} \quad (\text{A.13})$$

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