

## DESIRED TRAJECTORY GENERATION FOR BALANCING WITH RESPECT TO PERIODIC EXTERNAL FORCE

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This paper presents a desired trajectory generation of the biped base joint that keeps the CoP position at the constant position when periodic external forces are exerted. For a single sinusoidal external force, the desired trajectory is set based on the frequency transfer function that relates the desired angle and the external force to the ankle torque. For generic periodic external forces, Fourier expansion is introduced, and the Fourier coefficients are estimated based on the framework of an adaptive control. Simulations will demonstrate the effectiveness of this method.

### 1. Introduction

Biped robots are required to maintain the balance under disturbance. If a periodicity is observed in a disturbance as shown in Fig. 1, human learns this periodicity and make the best use of it for the balancing. From this point of view, we consider a learning of the desired trajectory that keeps the ZMP (zero moment point)[1], i.e., the CoP (center of pressure)[2], at the constant place using a simplified biped model based on an inverted pendulum that is sometimes utilized for the biped modeling [3,4].

As an introduction of biped locomotion, the balance control [5, 6] is an important problem to solve. The ZMP criterion is effectively utilized to locomotion planning [7, 8, 9]. There, the reference trajectories are basically calculated using the strict model of the robots, although real-time generations or online modifications of the reference trajectories are proposed to allow the robustness [10, 11, 12, 13, 14]. Some works treat the ZMP position as direct feedback signal to the torques [15, 16].

This paper aims at computing the reference trajectories for balancing by learning, which is not related to the robot dynamics or the effects from environments. For this purpose, an unknown periodic external force, which is expected to describe any robot dynamics or any environmental conditions, is introduced. Periodic external force is compatible to acting forces during locomotion: usually, locomotion consists of periodic movements that produce

periodical interaction force. It implies that, if the balancing method with respect to periodic external forces is established, it is also applicable to motion planning of the locomotion pattern, the planning by leaning simulation. This is a reason why we consider a balancing task under periodic external forces. An advantage of our method is the unnecessary of strict models if the external force is periodic.

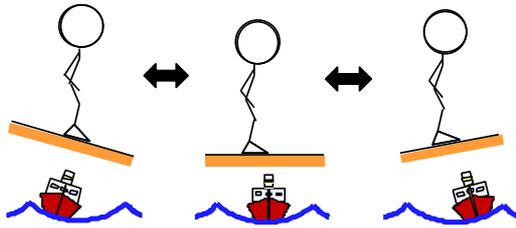


Fig. 1 Biped balance under periodic external force

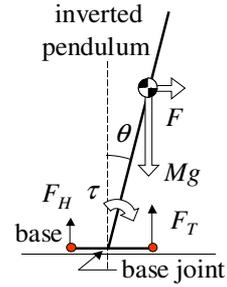


Fig.2. Simple biped model

## 2. Problem Formulation

A simplified biped balance model by inverted pendulum shown in Fig. 2 is utilized to analyze or simulate a control and learning method. This model consists of an inverted pendulum and a symmetrical base within the sagittal plane. The base joint (ankle joint) is located at the center of the base. This joint contains an actuator as well as a position and a velocity sensor. Un external force  $F$  is exerted at the CoM (center of mass) of the inverted pendulum to the horizontal direction. The motion equation of this model becomes

$$I\ddot{\theta} = MgL \sin \theta + FL \cos \theta + \tau \quad (1)$$

And the position of CoP,  $P_{CoP}$ , from the base center is given as

$$P_{CoP} = (F_T - F_H) / (F_T + F_H) \cdot \tau \quad (2)$$

Now, a simple PD control is applied to the balance control of this model:

$$\tau = K_d(\dot{\theta}_d - \dot{\theta}) + K_p(\theta_d - \theta) \quad (3)$$

Then, a problem that we consider in this paper is formulated as follows:

Problem: Suppose an external force is periodic. Then, design the desired trajectory  $\theta_d$  of the PD control (3) such that the position of the CoP should be kept at the bottom of the base joint, i.e.,  $P_{CoP} \equiv 0$ .

### 3. Methodology

#### 3.1. Outline

Generally, external forces are unknown. First of all, we assume that the periodic external force is completely known, especially, is described as a single sinusoidal function. Next, we will develop this method to a case where only the period of the external force is known. A way to deal with a completely unknown periodic external force is discussed as future works.

#### 3.2. A known sinusoidal external force

Consider the case where  $F$  is given as a single sinusoidal function

$$F = F_0 \sin \omega_0 t \quad (4)$$

Linearizing (1) around the upright posture and eliminating  $\theta$  by use of (3) after Laplace transformation, the transfer function to  $\tau$  is obtained.

$$\tau(s) = \frac{(K_d s + K_p)(I s^2 - MgL)}{I s^2 + K_d s + K_p - MgL} \theta_d(s) + \frac{-L(K_d s + K_p)}{I s^2 + K_d s + K_p - MgL} F(s) \quad (5)$$

Here, (2) indicates that, to keep the CoP at the bottom of the base joint,  $\tau(s) \equiv 0$  must be always satisfied. Thus, we should define  $\theta_d$ , depending on  $F(s)$ , as

$$\theta_d(s) = \frac{L}{I s^2 - MgL} F(s) \quad (6)$$

So, with respect to a periodic external force given by (3), set  $\theta_d$  as

$$\theta_d = \frac{-L F_0}{I \omega_0^2 + MgL} \sin \omega_0 t \quad (7)$$

then the CoP stays at the constant position.

#### 3.3. Periodic external force with known period

If period  $T_e$  is known, external force is described as the Fourier series with the base angular frequency  $\omega_e = 2\pi / T_e$ :

$$F = \sum_k^n \{ \alpha_k S_k + \beta_k C_k \} \quad (8)$$

Where  $S_k = \sin k \omega_e t$  and  $C_k = \cos k \omega_e t$ . Here, assume that  $n$  is sufficient large. Then, utilizing the method in the previous section,  $\theta_d$  should be given as

$$\theta_d = \sum_k^n \{ \theta_{sk} S_k + \theta_{ck} C_k \} \tag{9}$$

$$\theta_{sk} = \frac{-\alpha_k L}{I(k\omega_e)^2 + MgL}, \theta_{ck} = \frac{-\beta_k L}{I(k\omega_e)^2 + MgL} \tag{10}$$

Then, the unknown parameters  $\alpha_k$  and  $\beta_k$  can be obtained from an adaptive control which was proposed by Slotine and Li [17].

**4. Simulations**

An object of simulation is to examine the trajectory generation that keeps CoP at the constant position with respect to external force with known period. The model in Fig. 1 is used in this simulation, where  $I = 75.0 [kgm^2]$ ,  $M = 60.0 [kg]$ , and  $L = 1.0 [m]$ . An external force is set as

$$F = 25 \sin(2\pi \cdot \frac{1}{3} \cdot t) + 18 \sin(2\pi \cdot \frac{1}{4} \cdot t) \tag{11}$$

and the period 12s, the least common multiplier of two sinusoidal components 3s and 4s, was given to the controller as known parameter. Other control parameters are:  $K_p = 1200$ ,  $K_d = 700$  and  $n = 19$ .

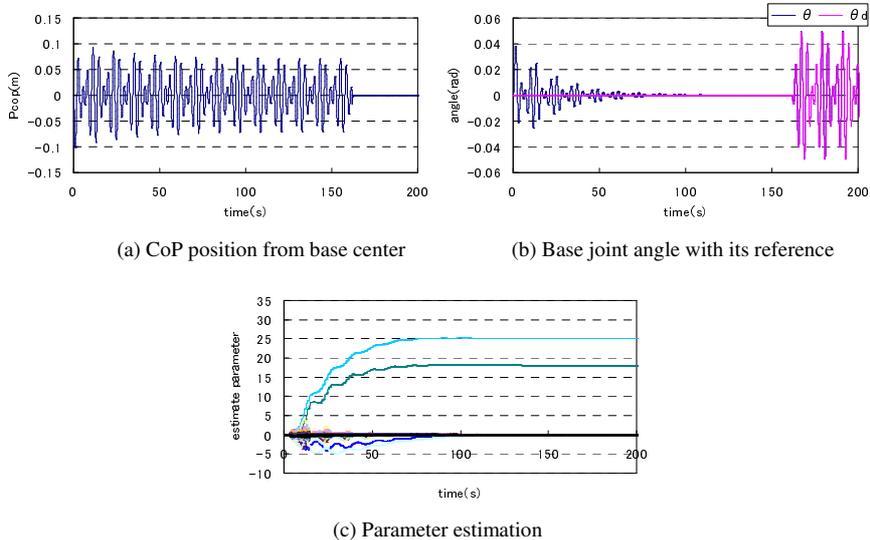


Fig. 3 Simulation results for periodic external force with known period

Fig. 3 (a) is the time-based plot of the CoP position, while Fig. 3 (b) is that of the desired angle. The period until around 160s is a learning phase where parameters on the external force were estimated under the PD control with  $\theta_d = 0$  [rad]. Fig. 3(c) is the time-based plot of parameter learning. Both sinusoidal components in (11), 25 and 18, are correctly acquired. After this period, the parameter learning was stopped, and the desired trajectory was switched using the desired trajectory construction (9) and (10) based on the learned parameters. In this phase, the CoP was always kept around 0m, by actuating the base joint so as to follow the periodic external force, which was observed the matched trajectories of the  $\theta$  and  $\theta_d$ .

## 5. Discussions and Conclusions

The simulation results demonstrate that our method achieved the desired trajectory generation that keeps the CoP at the center of the base. This method requires that the period of the external force must be known in advance. For completely unknown periodic external force, its period must be estimated. A method based on a local auto-correlation [18] may achieve this estimation.

A possible significance of our method is that it will be applicable to the locomotion pattern planning by learning even if the dynamics of robots are uncertain: in the balancing task, the effect of the robot dynamics can be represented as the unknown external force that is generated by the body, leg, and arm motions during the locomotion. The period of such motions is usually known in the designing stage of the motion pattern. From this point of view, the calculation problem of the robot dynamics with uncertain parameters are replaced as the learning process under unknown external forces that is actually applied to the real robot in a periodic manner. But, this idea is still conceptual. Simulations with multi-linked dynamics as well as experiments using inverted or biped robots are our future works to achieve the above our goal.

## References

1. M. Vukobratovic, B. Borovac, D. Surla and D. Stokic, *Biped Locomotion: Dynamics, Stability, Control and Application*. Springer. (1990).
2. A. Goswami, *Postural Stability of Biped Robots and the Foot-Rotation Indicator (FRI) Point*. The International Journal of Robotics Research, **18**, 523 (1999).
3. S. Kajita and K. Tani. *Experimental study of biped dynamic walking*. Control Systems Magazine, IEEE, **16**, 13 (1996).
4. K. Nishiwaki, S. Kagami, Y. Kuniyoshi, M. Inaba, and H. Inoue. *Online generation of humanoid walking motion based on a fast generation method*

- of motion pattern that follows desired ZMP.* IEEE/RSJ 2002 International Conference on Intelligent Robots and System, 2684, (2002).
5. S.N. Napoleon and M. Sampei. *Balance control analysis of humanoid robot based on ZMP feedback control.* Proceedings of the 2002 IEEE/RSJ International Conference on Intelligent Robots and Systems, 2437 (2002).
  6. S. Ito and H. Kawasaki. *Regularity in an environment produces an internal torque pattern for biped balance control.* Biological Cybernetics, **92**,241, (2005).
  7. J. Yamaguchi and A. Takanishi. *Development of a Leg Part of a Humanoid Robot - Development of a Biped Walking Robot Adapting to the Humans' Normal Living Floor.* Autonomous Robots, **4**, 369 (1997).
  8. K. Hirai, M. Hirose, Y. Haikawa, and T. Takenaka. *The development of Honda humanoid robot.* Proc. of 1998 IEEE International Conference on Robotics and Automation, 1321 (1998).
  9. K. Mitobe, G. Capi, and Y. Nasu. *Control of walking robots based on manipulation of the zero moment point.* Robotica, **18**, 651 (2001).
  10. K. Nagasaka, M. Inaba, and H. Inoue. *Dynamic walking Pattern Generation for a Humanoid Robot based on Optimal Gradient Method.* Proc. of 1999 IEEE International Conference on Systems, Man, and Cybernetics, **6**, VI-908 (1999)
  11. Q. Huang, K. Kaneko, K. Yokoi, S. Kajita, T. Kotoku, N. Koyachi, H. Arai, N. Imamura, K. Komoriya, and K. Tanie. *Balance control of a biped robot combining off-line pattern with real-time modification.* Proc. of the 2000 IEEE International Conference on Robotics and Automation, 3346 (2000).
  12. T. Sugihara, Y. Nakamura, and H. Inoue. *Real-time humanoid motion generation through ZMP manipulation based on inverted pendulum control.* Proceedings of 2002 IEEE International Conference on Robotics and Automation, 1404 (2002).
  13. D. Wollherr and M. Buss. *Posture modification for biped humanoid robots based on Jacobian method.* Proceedings. 2004 IEEE/RSJ International Conference on Intelligent Robots and Systems, 124, 2004.
  14. V. Prahlad, G. Dip, and C. Meng-Hwee. *Disturbance rejection by online ZMP compensation.* Robotica, **26**, 9, 2007.
  15. P. Kulvanit, H. Wongsuwan, B. Srisuwan, K. Siramee, A. Boonprakob, T. Maneewan, and D. Laowattana. *Team KMUTT: Team Description Paper.* Robocup 2005: Humanoid League, 2005.
  16. S. Ito, S. Amano, M. Sasaki, and P. Kulvanit. *A ZMP Feedback Control for Biped Balance and its Application to In-Place Lateral Stepping Motion.* Journal of computers, **3**, 23, (2008).
  17. J.J. Slotine, W.Li, *Applied nonlinear control.* Prentice-Hall (1991).
  18. S. Ito, T. Kashima, M. Sasaki, *Engineering Applications of Artificial Intelligence*, doi:10.1016/j.engappai.2010.04.004 (in printing)