Proceedings of the 2nd International Symposium
on Adaptive Motion of Animals and Machines,
Kyoto, March.4-8, 2003

Static balance control and external force estimation using ground reaction forces

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Abstract

In this paper, we consider a balance control with focusing on ground reaction forces. As an example of balance control, the static balance with constant external forces acting is treated. Ankle joint torque is used to control the balance, which is defined as a PD control with ground reaction force feedback. Then, the stationary posture emerges such that changes with external forces. Furthermore, the external forces can be estimated from the states of stationary posture. Based on these estimates, a desired posture appropriate to the current environments is internally generated, which is utilized for the behavior at the next trail.

1. Introduction

Locomotion necessary produces interactions with environments. Walking on the ground, the foot contacts it, which returns reaction forces. The forces, i.e., the ground reaction forces vary with terrain conditions as well as disturbances and executing motions. In other words, all the responses of the action to environments, or the changes there, are reflected in the ground reaction forces. Therefore, the ground reaction forces are informative for locomotion control. To measure and control them will be an efficient method for keeping the balance.

As one of the indicators for planning the successful locomotion with keeping balance, the ZMP (zero moment point) [1] criterion is proposed. The ZMP is the point on the ground surface around which the horizontal moment generated by inertial and gravitational forces becomes zero. If the ZMP stays at the inside of the convex full of footprints, the foot part does not rotate, implying that the tumbling does not occur. In the conventional control of biped robots, the desired motion is previously calculated based on the ZMP criterion [1, 2] and the positional control is performed to track it. However, if there exist disturbances from environments or the parameter errors in the motion planning, the ZMP does not always come to the designed position. In the recent researches [3, 4, 5, 6], the ground reaction forces are measured to compute the actual ZMP position, and are utilized to keep balance. The efficiency of their methods are demonstrated experimentally by robot performances. However, the theoretical analysis is not sufficient because of the complexity of robot dynamics.

To keep the analysis simple, we consider only the static balance in this paper. The static balance should be a base of the dynamic balance and its adequate analysis is fundamental to locomotion. Especially, for the reason we mentioned first, we focus on ground reaction forces. When we get on the train or bus with standing there, we feel through the sole of our feet that the gravity point is moving at the start and stop, or curves. This phenomenon is described with the term CoP (center of pressure). According to [7], the CoP is defined as the point on the ground where the resultant of ground reaction force acts. We here hypothesize that the balance control is achievable by the CoP control, and so propose a control method of ground reaction forces that approximates a CoP feedback control. Furthermore, in order to treat environmental changes, we consider the simplest case where the constant external forces are exerted. Under these situations, the external forces represent the environmental conditions. This case study contains standings on the slope where the direction of gravity is not orthogonal to the ground surface. Because the environmental conditions are reflected in the ground reaction forces, the parameters of environment, i.e., the magnitude and direction of the external forces can be estimated from the stationary state. Such an acquisition of the environmental information through the motion will be useful for the feedforward execution of motions.

2. Standing in environment with constant external force

2.1. Problem

Humans sometimes adjust their upright posture using only ankle joints, which is called “ankle strategy”[8]. Even in such a simple task, we can observe adaptive behaviors with respect to their environmental conditions. For example, when standing on a slope, humans adjust their ankle joints in such a way that the body is always oriented in the gravitational direction. Besides, standing in the strong wind, humans lean their body upward against the wind. These two examples are the same in that external forces act in the static standing. Here, we regard these unknown constant external forces as environmental states.

To achieve such an adaptive behavior, the following problem arises: the desired posture cannot be determined until the environmental states are given. To treat this problem, not the control of position but the control of force (especially the ground reaction forces) is essential. Under this idea, the stationary posture naturally results from the balance control based on the ground reaction forces. Firstly, we introduce a control law with the feedback of ground reaction forces [9]. Next, we explain a manner whereby the external forces exerted are estimated.

2.2. Model and assumptions

When static balance is controlled by the ankle strategy [8], the biped system can be represented as the two link system consisting of body part and foot part, as shown in Fig. 1. According to the symmetry in the lateral direction, we consider only the one side. These two links are connected at the ankle joint, and torque for balance control can be generated here. For the sake of simplicity, the motion of this model is restricted to the sagittal plane on level ground. The model contacts the ground only at the two points of the foot, i.e., toe and heel. Here, the vertical component of ground reaction forces F_T (at the toe) and F_H (at the heel) are detectable. To make the calculation simple, we assume a symmetrical foot part and the low ankle joint position. According to the former assumption, we put the distance from ankle joint to heel or toe to the same value ℓ. The latter assumption implies that the vertical force form the body part does not cause the moment against the foot part.

If the friction on the ground is large enough and the balance is kept successfully, the foot part does not make a motion. Thus, we only consider the motion

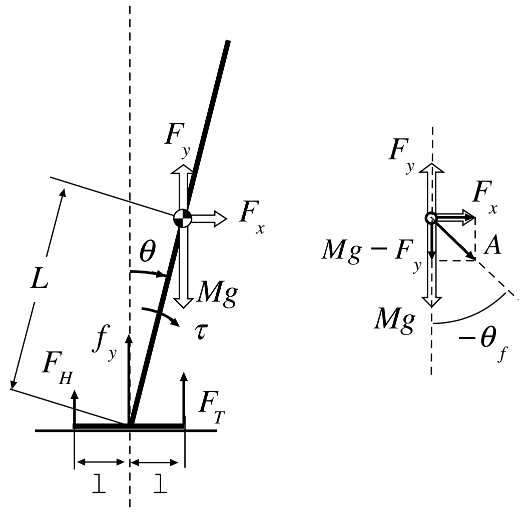


Figure 1: Link model.

of the body part. In order to treat an adaptive behavior with respect to the environment, it is important to include the external forces in the motion equation. The motion equation of the body part with the external forces can be described as follows,

$$I\ddot{\theta} = MLg\sin\theta + F_xL\cos\theta - F_yL\sin\theta + \tau, \quad (1)$$

where M is a mass of the body part, I is an inertial moment of the body part around the ankle joint, L is the length between ankle joint and the COG of the body part, θ is the ankle joint angle from the vertical direction, τ is the ankle joint torque, F_x and F_y are constant external forces, respectively, in the horizontal and vertical directions, and g is gravitational acceleration. To the convenience in the analysis, we transform the motion equation (1) as follows:

$$\begin{aligned} I\ddot{\theta} &= (Mg - F_y)L\sin\theta + F_xL\cos\theta + \tau \\ &= AL\sin(\theta - \theta_f) + \tau \end{aligned} \quad (2)$$

where

$$A = \sqrt{(Mg - F_y)^2 + F_x^2} \quad (3)$$

and θ_f is a constant which satisfies these equations,

$$\sin\theta_f = -\frac{F_x}{A}, \ \cos\theta_f = \frac{Mg - F_y}{A}. \quad (4)$$

Note that A as well as θ_f depend on environmental states, i.e.,F_x and F_y.

2.3. A control law

If the static balance is maintained, the foot part does not rotated around heel or toe. This condition is de-

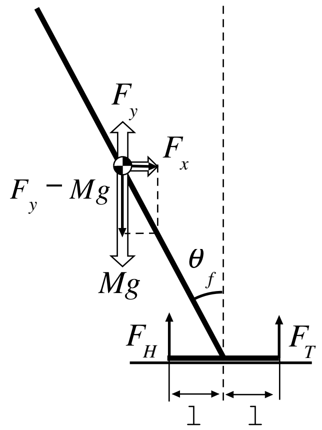


Figure 2: stationary posture.

scribed as $F_T > 0$ and $F_H > 0$. Taking the stability margin [10] into account, it is favorable that the body mass is evenly weighted to the toe and heel, i.e., $F_T = F_H$. The relation between ankle torque τ and ground reaction forces F_T/F_H is given from the balance of moment around toe or heel,

$$F_T = -\frac{1}{2\ell}\tau + \frac{1}{2}m + \frac{1}{2}f_y, \tag{5}$$

$$F_H = \frac{1}{2\ell}\tau + \frac{1}{2}m + \frac{1}{2}f_y. \tag{6}$$

Here, m is a mass of the foot part, and f_y is the vertical force from the body part, which is given by

$$f_y = -ML\ddot{\theta}\sin\theta - ML\dot{\theta}^2\cos\theta + Mg - F_y. \tag{7}$$

From (5) and (6), $F_T = F_H$ can be satisfied if we set $\tau = 0$. However, this makes the body part fall down. Accordingly, the goal of control law is to make F_H and F_T equal at the stationary state with keeping the body part from tumbling.

To achieve this goal, the following control law is proposed:

$$\tau = -K_d\dot{\theta} + K_p(\theta_d - \theta) + K_f\int(F_H - F_T)dt. \tag{8}$$

where K_d, K_p and K_f are feedback gains. The meaning of each term is as follows: The first and the second terms of the left hand side are just PD control, which act to prevent the body part from falling down for the moment. The last term eliminates the difference between ground reaction forces F_T and F_H at the standing posture.

2.4. Stationary posture

Here, we analyze the stationary posture for the control law (8). Before that, we introduce a new state variable τ_f which is defined by

$$\tau_f = \int(F_H - F_T)dt, \tag{9}$$

Then, the control law is described as

$$\tau = -K_d\dot{\theta} + K_p(\theta_d - \theta) + K_f\tau_f. \tag{10}$$

Furthermore, subtracting (5) and (6), we obtain the relation between $F_H - F_T$ and τ,

$$F_H - F_T = \frac{1}{\ell}\tau. \tag{11}$$

Because the left hand side of this equation is equal to $\dot{\tau}_f$, the above equation turns to

$$\dot{\tau}_f = \frac{1}{\ell}\left\{-K_d\dot{\theta} + K_p(\theta_d - \theta) + K_f\tau_f\right\}. \tag{12}$$

From (2) and (10), on the other hand, the next equation is obtained,

$$I\ddot{\theta} = AL\sin(\theta - \theta_f) - K_d\dot{\theta} + K_p(\theta_d - \theta) + K_f\tau_f \tag{13}$$

From the above two equations, the stationary state can be calculate by putting $\dot{\theta} = \ddot{\theta} = \dot{\tau}_f = 0$. As the result of calculation, the stationary posture is given as

$$(\bar{\theta}, \bar{\tau}_f) = (\theta_f, -\frac{K_p}{K_f}(\theta_d - \theta_f)) \tag{14}$$

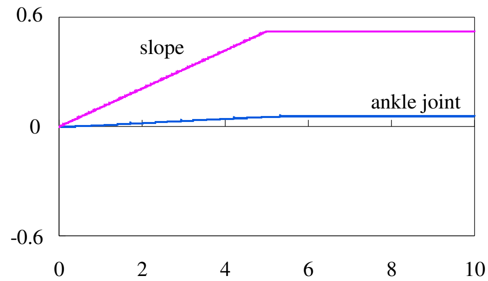
At the stationary state illustrated in Fig. 2, the torque of ankle joint is zero, i.e., $\tau = 0$, since the external and gravitational forces are balanced. In addition, from (5) and (6), the ground reaction forces F_H and F_T become the same, and so the goal of control is achieved.

The stability of the stationary state (14) is ensured locally by appropriate feedback gains, as shown in appendix.

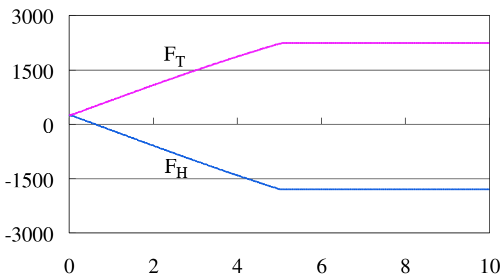
2.5. Estimation of external force

It should be noted that the stationary posture is independent of θ_d, the desired value of the PD control part in (8). It only depends on the environmental states F_x and F_y. Here, we consider a role of θ_d.

Suppose that there are no interactions with the environment, e.g., the foot part does not contact the ground. Then, $F_H = F_T = 0$ and so the ankle joint tends to be θ_d according to (8). If the foot part begins to contact

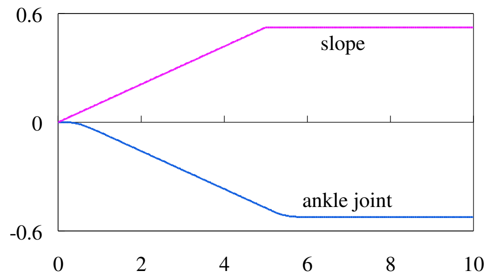


(a) slope angle α and ankle joint angle θ.

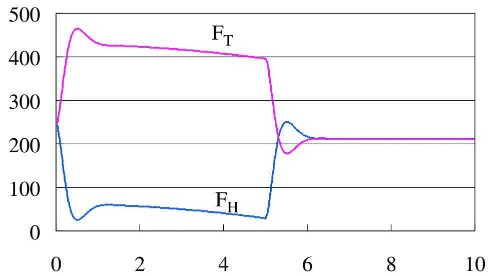


(b) ground reaction forces F_H and F_T.

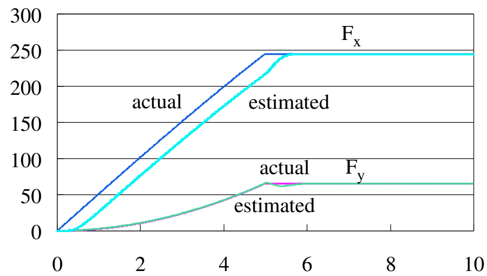
Figure 3: Simulation with PD control ($K_f = 0$).



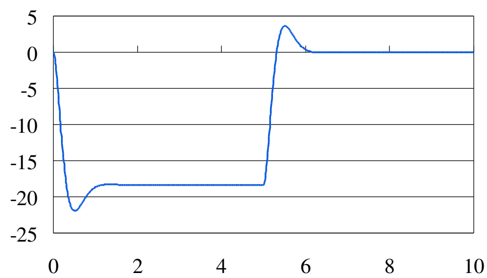
(a) slope angle α and ankle joint angle θ.



(b) ground reaction forces F_H and F_T.



(c) Estimation of External forces.



(d) ankle joint torque τ.

Figure 4: Simulation of PD control with ground reaction force feedback ($K_f = 1$).

the ground and the interactions start, the θ_d becomes the initial state of the dynamics.

It is favorable that the initial state, i.e., θ_d should be set near the stationary one which is determined by the external forces, because the stability is ensured only locally around the stationary state. But, to set the appropriate initial state, the external forces have to be known a priori. If so, the θ_d can be set as

$$\theta_d = \arctan \frac{\hat{F}_x}{\hat{F}_y - Mg} \tag{15}$$

where $\hat{F}_x$ and $\hat{F}_y$ are the estimate values of external forces.

Unless the environments drastically change in the short time, the estimation of the environments will be executable based on the results of previous or current actions. For the problem setting in this paper, the stationary states emerging from the control law (8) contains abundant information on the environments, because the stationary posture depends heavily on the external forces. Using them, i.e., the stationary state of the posture $\bar{\theta}$ and ground reaction force $\bar{F}_T$, $\bar{F}_H$, the environments can be estimated as

$$\hat{F}_y = (M + m)g - (\bar{F}_H + \bar{F}_T) \tag{16}$$

$$\hat{F}_x = -(\bar{F}_H + \bar{F}_T - mg)\tan\bar{\theta} \tag{17}$$

These estimates will be useful to perform motions in the feedforward manner at the next trail.

2.6. CoP feedback control

The control law (8) is approximately regarded as a feedback control of CoP. Around the CoP, the moment generated by vertical component of ground reaction forces equals to zero. Using this property, the position of CoP, i.e., P_{CoP} is expressed by

$$P_{CoP} = \frac{F_T - F_H}{F_T + F_H}\ell \tag{18}$$

where the origin of CoP coordinates is set to the ankle joint. The denominator $F_T + F_H$ represents mainly the total weight and so doesn't change so much at the static balance. Therefore, defining $K'_F = \ell/(F_T + F_H)$ as constant, the above equation becomes

$$P_{CoP} = K'_F(F_T - F_H) \tag{19}$$

and (8) turns to

$$\tau = -K_d\dot{\theta} + K_p(\theta_d - \theta) + K_F \int (P_d - P_{CoP})dt. \tag{20}$$

where $K_F = K_f/K'_F$, and P_d is the desired position of CoP which is zero in (8).

3. Simulation

In computer simulations, we set parameters as M=50, m=0.1, L=0.75, $\ell = 0.1$, $I = ML^2/4$, $\theta_d = 0$, initial states as $\theta(0) = \dot{\theta}(0) = 0$, and feedback gains as K_d = 1200, K_p=3500. The external forces are given by the next equations:

$$F_x = Mg\sin\alpha, F_y = Mg(1 - \cos\alpha) \tag{21}$$

which corresponds to the external forces exerted on the slope with the gradient α. The gradient α is changed according to the equation,

$$\alpha = \begin{cases} \frac{\pi}{6}\frac{t}{5} & (0 < t < 5) \\ \frac{\pi}{6} & (5 \leq t) \end{cases} \tag{22}$$

The ground reaction forces are calculated assuming that they can take negative values. For the computation, 4-order Runge-Kutta method is used with time step 0.001. The period of simulating time is 10.

Two cases are examined: (i) only the PD control ($K_f = 0$), and (ii) the PD control with ground reaction force feedback ($K_f = 1$). In the case (i), the ankle joint is kept near 0 because of the high-gain PD control, as illustrated in Fig. 3(a). However, the ground reaction force F_H takes negative values when the slope becomes steep as shown in Fig. 3(b), implying that the tumbling actually occurs. In the case (ii), however, the ankle joint angle changes with slope gradient as illustrated in Fig. 4(a), resulting that the ground reaction forces never take negative values as shown in Fig. 4(b). Furthermore, as depicted in Fig. 4(c), the external forces are correctly estimated. Fig. 4(d) shows the ankle joint torque, indicating that it becomes zero at the stationary state. These results represents the effectiveness of the ground reaction force feedback for the static balance control.

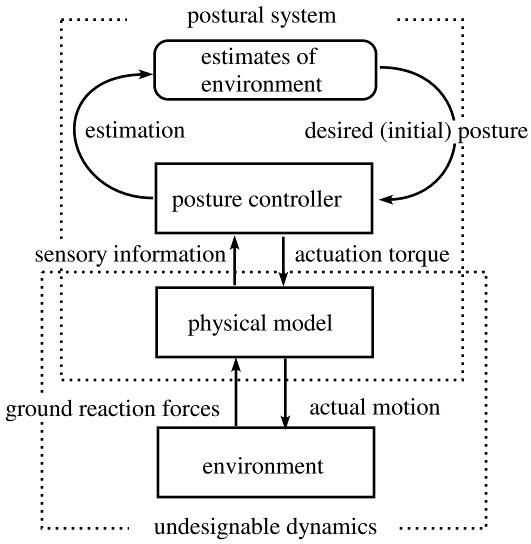


Figure 5: Control scheme.

4. Conclusion

The control scheme proposed in this paper is summarized as Fig. 5, whose original model will be found in [11]. In the conventional method, the control of locomotion is achieved using only the middle loop: the sensory information (mainly joint angles) is used to generate the actuation torque. The desired posture provided from the upper level does not change according to the situations.

In order to treat the environmental variations, the interactions with the environment are important. This is fulfilled at the lowest loop in Fig. 5. In the static balance control, the influences from environments are obtained through the ground reaction forces. Adaptive behaviors to the environmental conditions are feasible by making good use of them.

Because the ground reaction forces contain much external information from environments, they are also available to estimate their conditions. Based on the estimates, the motion patters favorable to the current environments are internally generated. The highest loop in Fig. 5 is responsible to such functions. A kind of intelligence on the adaptability may exist in the internal generation of the desired posture based on the estimates of environments through this loop.

Appendix

Here, we examine the stability of the equilibrium point given by (14). Differential equations linearized at the equilibrium point are,

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\tau}_f \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{AL-K_p}{I} & -\frac{K_d}{I} & \frac{K_f}{I} \\ -\frac{K_p}{\ell} & -\frac{K_d}{\ell} & \frac{K_f}{\ell} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \tau_f \end{bmatrix} \quad (23)$$

where $\theta_1 = \theta$ and $\theta_2 = \dot{\theta}$. The characteristic equation of this linear system is given by

$$\lambda^3 + p_2\lambda^2 + p_1\lambda + p_0 = 0 \quad (24)$$

where

$$p_2 = \frac{K_d\ell - K_f I}{I\ell}, p_1 = \frac{K_p - AL}{I}, p_0 = \frac{K_f AL}{I\ell} \quad (25)$$

According to the method formulated by Routh/Hurwitz, the necessary and sufficient conditions that the equilibrium point becomes stable are given as

$$p_0 > 0,\ p_1 > 0,\ p_2 > 0,\ p_1 p_2 - p_0 > 0 \quad (26)$$

From these inequalities, if the feedback gains satisfy

$$K_p > AL > 0 \quad (27)$$

$$\frac{\ell}{I} K_d > K_f > 0 \quad (28)$$

$$(K_d\ell - K_f I)K_p > K_d \ell AL \quad (29)$$

the local stability of the equilibrium (14) is ensured.

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