In-place lateral stepping motion of biped robot adapting to slope change

Satoshi Ito[*,†], Shinya Amano[*] Minoru Sasaki[*] and Pasan Kulvanit[‡]
[*] Faculty of Engineering, Gifu University, Gifu 501-1193, Japan
Email: satoshi@gifu-u.ac.jp
[†] BMC Research Center, RIKEN, Nagoya 463-0003, Japan
[‡] Institute of Field Robotics (FIBO), King Mongkut's University of Technology Thonburi, Bangkok, Thailand

Abstract— **A zero moment point (ZMP) criterion is a powerful method for biped control. Although many works plan locomotion patterns based on it, the ZMP is not always controlled in a feedback manner. We proposed a static balance control based on a feedback of ZMP positions, and applied it to a weight shift motion in biped double support phase. In this paper, we extend these methods to the biped in-place stepping motion. We examine the effectiveness of this control method by not only simulations but also robot experiments.**

I. Introduction

The concept of the zero moment point (ZMP) is proposed to evaluate the dynamic stability of biped locomotion [1]. Many studies of biped walking utilized the so-called ZMP criterion to stabilize biped locomotion. Initially, the reference trajectories of each joint angle, or the center of mass (CoM) of the body, are planned so that the computed ZMP, obtained as a cross point of the inertial and gravitational force resultant vector and the ground, stays inside the foot support. The joint angles, or the CoM of the body, are then controlled in a feedback manner to follow these reference trajectories. In this control strategy, however, the ZMP is not controlled directly since the actual position of the ZMP is not measured during the locomotion. This implies that the modeling error such as irregularities of the ground surface may disturb the ZMP trajectory from the reference one. Some studies introduced the concept of the ZMP feedback [2], [3], but the analysis of the dynamics was not sufficient to ensure the stability. Other works proposed an on-line trajectory planning based on ZMP position [4], [5].

In the case of lateral control during biped stepping, by directly selecting the ZMP as a control variable, the balance may be maintained in any environmental conditions without modifying the reference trajectories. Goswami[6] states that the ZMP is identical to the CoP. From the viewpoint of static balance, a method that utilizes feedback of the position of the CoP (Center of Pressure) is proposed by [7] and [8]. This method is applied to a weight shift motion by making the reference position of CoP time-varying [9]. Based on this idea, we here propose a motion control method for lateral in-place stepping in which the motor planning is not effected from the irregularities of the ground. The nature of the lateral stepping is to move the CoP position from one foot to the other foot. This nature is invariant even if the environmental conditions, such as the gradient of the ground, change. On the other hand, the desired trajectory of the joint angles always varies with it, as shown in Fig. 1. This is the reason why we select the CoP as a control variable. A complete walking gait is composed of sagittal plane inverted pendulum tumbling motion and lateral plane motion. The control principle in the sagittal plane differs from that in the lateral plane. We, here, restrict our scope to the stepping control in the lateral direction by considering only the CoP shifting motion.

This paper is structured as follows. In section II, we review a fundamental approach for the balance using the CoP position feedback. In section III, we construct a control strategy for stepping motion based on the method reviewed in section II, and show computer simulation results. In section IV, we give details of our experimental apparatus and presents experiment results of the in-place lateral stepping motion. We then give concluding remarks in section V.

II. Fundamental theory

A. Balance control by CoP regulation

The lateral stepping control, we propose here, is an expansion of the method using the CoP position feedback [8], [9]. This method successfully maintains static balance under unknown external force.

With this method, the followings are assumed. An inverted pendulum (body segment) with a supporting segment (foot segment) is used as a model of biped balance. Two segments are connected at the ankle joint as shown in Fig. 2(left). The motion occurs only within the x-y plane. The ankle joint angle θ and its velocity $\dot{\theta}$ are detectable. They are used to drive a torque τ at the ankle joint. The foot segment contacts the ground at two points (heel and toe), where the vertical components of the ground reaction force (F_H and F_T) are also detectable. The foot segment does not slip on the ground and its shape is symmetrical in the anterior-posterior direction. The ankle joint is located in the middle of the foot segment with zero height.

The environmental condition is represented as an unknown constant external force that is exerted to the CoM of the pendulum, whose horizontal and vertical component is F_x and F_y, respectively. We will see later that the slope change can be well described by the use of F_x and F_y. If the balance is maintained, the equation of the motion of the inverted pendulum is as follows.

$$\begin{aligned} I\ddot{\theta} &= MLg\sin\theta + F_xL\cos\theta - F_yL\sin\theta + \tau. \\ &= AL\sin(\theta - \theta_f) + \tau \end{aligned} \quad (1)$$

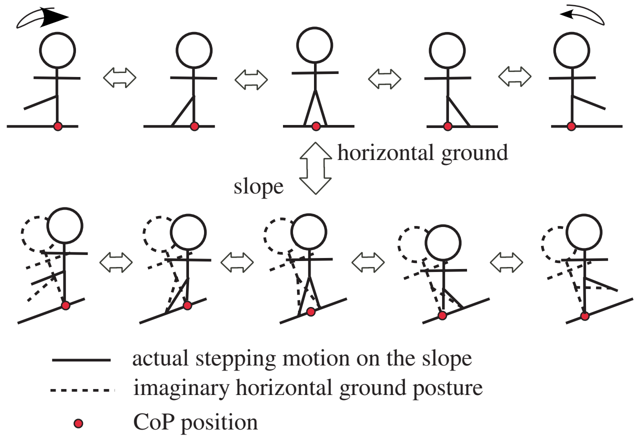


Fig. 1. Lateral stepping motion on different gradient condition.

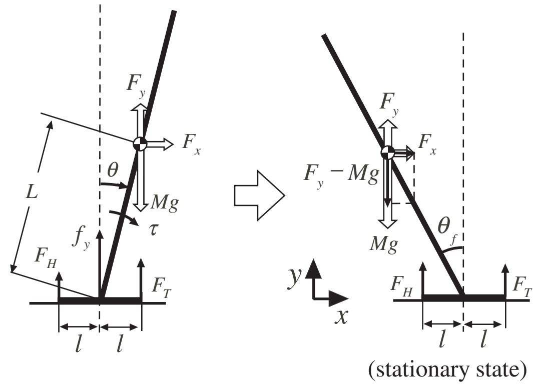


Fig. 2. Link model for static balancing.

where

$$A = \sqrt{(Mg - F_y)^2 + F_x^2} \tag{2}$$

and θ_f is a constant satisfying

$$\sin\theta_f = -\frac{F_x}{A}, \;\; \cos\theta_f = \frac{Mg - F_y}{A}. \tag{3}$$

Here, M is a mass of the inverted pendulum. I is its moment of inertia around the ankle joint. L is the length between ankle joint and the center of mass (CoM) of the pendulum. g is the gravitational acceleration.

The ankle joint torque gives an effect to the ground reaction forces. Their relations are described as

$$F_T = -\frac{1}{2\ell}\tau + \frac{1}{2}mg + \frac{1}{2}f_y, \tag{4}$$

$$F_H = \frac{1}{2\ell}\tau + \frac{1}{2}mg + \frac{1}{2}f_y. \tag{5}$$

Here, m is a mass of the foot segment. ℓ is the length from the ankle joint to the toe or the heel. f_y is the vertical component of the force from the inverted pendulum.

The effect of ground reaction forces is unilateral. They always push the foot segment against the gravitational direction, implying that F_T and F_H must be kept positive for preventing the foot segment from rotating. Moreover, F_T and F_H should be not only positive but also equal from the viewpoint of stability margin of static balance since the CoP is located in the middle of the foot segment in this situation. Note, however, that the CoP at this position yields maximum static stability margin, therefore, the tumbling can only occur with the maximum moment.

To achieve $F_T = F_H$ at the stationary state, we proposed the following control method containing the feedback of the ground reaction forces:

$$\tau = -K_d\dot{\theta} + K_p(\theta_d - \theta) + K_f\int(F_H - F_T)dt. \tag{6}$$

The stationary posture $\bar{\theta}$ is given as $\bar{\theta} = \theta_f$. At the same time, $F_H = F_T$ is satisfied for any constant external force F_x and F_y (as shown in Fig. 2). Then, the CoP is controlled to the middle of the foot segment. The asymptotic stability of this steady state is locally ensured for appropriate feedback gains K_d, K_p and K_f [8].

The control law (6) is equivalent to the feedback control of CoP. The resultant moment of the vertical component of all the ground reaction forces become zero around the CoP. Using this property, the CoP position is calculated as follows,

$$P_{CoP} = \frac{F_T\ell - F_H\ell}{F_T + F_H}, \tag{7}$$

where, P_{CoP} is the horizontal position of CoP from the ankle joint. If the motion is slow, $F_T + F_H$ represents total weight of the link system and thus can be regarded as constant. Defining constant K_w as

$$K_w = \frac{\ell}{F_T + F_H}, \tag{8}$$

the above equation changes to

$$P_{CoP} = -K_w(F_H - F_T). \tag{9}$$

Using this relation, (6) can be written as a form of the CoP position feedback

$$\begin{aligned} \tau \;\; = \;\; & -K_d\dot{\theta} + K_p(\theta_d - \theta) \\ & + K_f'\int(P_d - P_{CoP})dt. \end{aligned} \tag{10}$$

This equation has been extended to include P_d, the reference position of the CoP (see also sec. III), that was set to zero in (6). Here, $K_f' = K_f/K_w$ is also constant.

B. Weight shift movement by time-varying CoP reference.

In this section, the control method in the previous section is extended to a weight-shift movement in the double support phase of the biped locomotion [9].

This movement can be regarded as a tracking control of the CoP position. Indeed, each joint angle can be controlled to achieve such a weight shift movement. However, the joint trajectories will vary with the gradient of the ground. For example, the tilt angle of the whole body on the slope changes even in the static balance, not to mention the case of weight shift motion. The reference trajectories of the joint angles should be modified with variable environmental conditions, implying that joint angles are not appropriate as

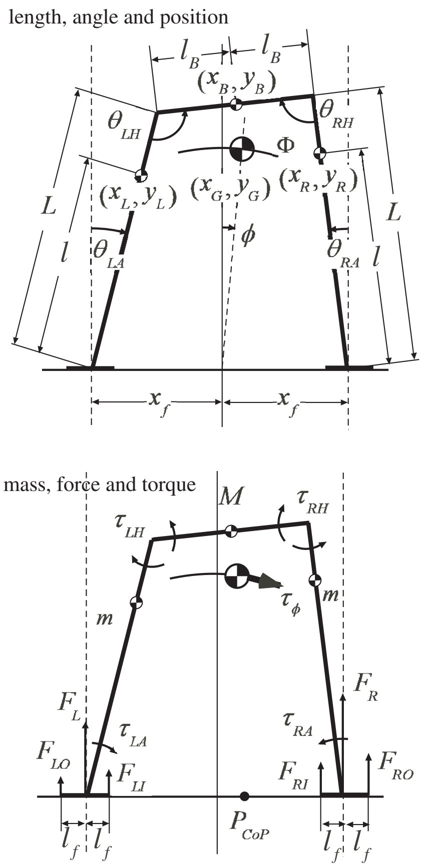


Fig. 3. Link model for motion control in double support phase .

controlled variables. On the other hand, the trajectory of the CoP, moving from one side to the other under the feet is invariant. Because of the invariance of the CoP trajectory regardless of the gradient of the ground, the CoP is selected as a controlled variable. Making the reference trajectory of the CoP, which was constant in the static balance control, to be time-varying, the weight shift motion is expected to be realized with no effect of the environmental conditions.

We assume that the lateral motion approximately describe the weight shift movement in the double support phase and so we restrict the motion within the lateral plane, which enable us to use a link model as shown in Fig. 3. This 5-link model consists of one body, two legs and two feet. In the weight shift movement, the flexion of knee joints is assumed to be small, hence, each leg is represented with only one link without knee. Ankle joints are assumed to be located at the center of the foot with zero height. At the end of both sides, the foot contacts the ground, and the ground reaction forces are detectable. Furthermore, angular positions and angular velocities are measurable at the ankle and the hip joints. Every joints actively generate torques.

Comparing to the problem establishment of the static balancing in the previous section, there are three different points - the number of contact points, the noncircular orbit of CoM motion, and the redundancy of actuators.

To cope with the first difference, we adopt an idea of the control law (10) equivalent to the CoP feedback. Around the CoP, the resultant moment of the vertical component of all the ground reaction forces become zero. In double support phase, the position of the CoP denoted by P_{CoP} is calculated from the vertical component of ground reaction forces at four contact points, i.e., F_{RO}, F_{RI}, F_{LO}, and F_{LI} as follows.

$$P_{CoP} = -\frac{F_{RO}}{F_{all}}(x_f+\ell_f) - \frac{F_{RI}}{F_{all}}(x_f-\ell_f) + \frac{F_{LI}}{F_{all}}(x_f-\ell_f) + \frac{F_{LO}}{F_{all}}(x_f+\ell_f) \quad (11)$$

$$F_{all} = F_{RO}+F_{RI}+F_{LI}+F_{LO}. \quad (12)$$

Here, the subscript $_{RO}$, $_{RI}$, $_{LI}$ and $_{LO}$ distinguish the four contact points: the right outside, the right inside, the left inside and the left outside respectively. ℓ_f is the length from the ankle joint to each side of the foot segment. x_f is the half distance between two ankle joints.

The second difference is the orbit of the CoM. The CoM motion of static balance model in Fig. 2 follows a circular arc orbit, while the one in the Fig. 3 is not always so. Thus, we define the new coordinate frame to describe the trajectory of the CoM as shown at the top of Fig. 3. The coordinate of the CoM in this frame is represented by ϕ. It is preferable that the new coordinate frame is naturally extended from the one for the single support phase. From this point of view, we define ϕ as a sway angle of the CoM from the direction vertical to the ground.

$$\phi = \arctan 2(x_G, y_G). \quad (13)$$

Here, (x_G, y_G) denotes the coordinate of the CoM whose origin is set in the midpoint of the two ankle joints. The control law is defined at this coordinate frame. The generalized force τ_ϕ for the CoP regulation is defined as the force which is exerted in the tangential direction of the orbit, and, following (10), given as

$$\tau_\phi = -K_d\dot{\phi} + K_p(\phi_d - \phi) + K_f\int(P_d - P_{CoP})dt \quad (14)$$

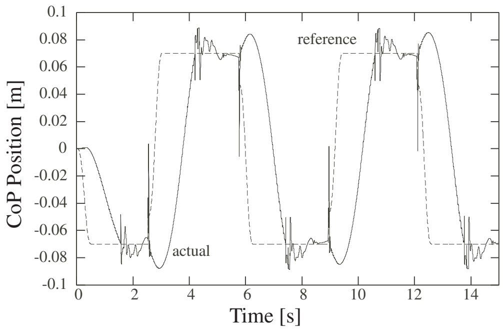
Here, P_{CoP} is calculated from (11). Since this equation takes the same form as (10), thus as (6), the CoP is expected to converge to the reference value.

The third difference is the redundancy of the joint torque, To cope with this difference, we implemented the torque distribution scheme by solving Jacobian matrix equation. The Jacobian matrix in parallel link system is given as follows.

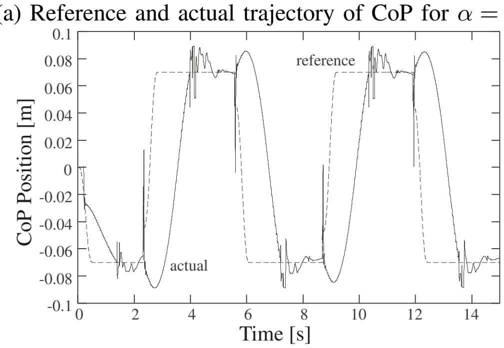
$$\Delta\boldsymbol{\theta} = \boldsymbol{J}(\boldsymbol{\theta})\Delta\phi. \quad (15)$$

where $\Delta\phi$ is a deviation of CoM along the orbit and $\Delta\boldsymbol{\theta}$ ($\boldsymbol{\theta} = [\theta_{RA}, \theta_{RH}, \theta_{LH}, \theta_{LA}]^T$) is the one of the joint angles. The subscript $_{RA}$, $_{RH}$, $_{LH}$ and $_{LA}$ represent the joint positions at the right ankle, the right hip, the left hip and the left ankle, respectively. From the principle of virtual work, the next relation holds between the generalized forces τ_ϕ and the joint torques $\boldsymbol{\tau} = [\tau_{RA}, \tau_{RH}, \tau_{LH}, \tau_{LA}]^T$,

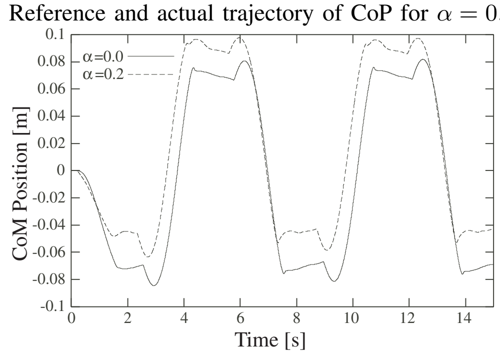
$$\tau_\phi = \boldsymbol{J}^T(\boldsymbol{\theta})\boldsymbol{\tau} \quad (16)$$



(a) Reference and actual trajectory of CoP for $\alpha = 0$



(b) Reference and actual trajectory of CoP for $\alpha = 0.20$



(c) Horizontal position of CoM.

Fig. 4. Simulation results.

Solving this equation, the joint torques are given by

$$\boldsymbol{\tau} = (\boldsymbol{J}^T(\boldsymbol{\theta}))^* \tau_\phi + (I - (\boldsymbol{J}^T(\boldsymbol{\theta}))^* \boldsymbol{J}^T(\boldsymbol{\theta}))\boldsymbol{p} \tag{17}$$

Here, $*$ denotes its generalized inverse matrix. $\boldsymbol{p}$ is an arbitrary 4-dimensional vector.

In [9], the authors examine the stability for the static CoP reference in this control method and show its effectiveness by applying it to control the weight shift movement under unknown external forces.

III. Control for the Stepping motion based on the CoP feedback

In this study, we consider a control method for stepping motion that is robust to an unknown constant external disturbance. The control methods for the weight shift motion as well as the single leg static balance in the previous section can be used in combination to automatically adjust the posture or motion with respect to the unknown external force and realize stepping motion using a simple biped robot. The main problem is how to switch the control law between the two schemes. Focusing on this aspect, we propose below a control method for stepping motion based on the CoP information.

A. Control in double support phase

At the start of walking, the legs are on the ground in the double support phase. To take the first step, the weight has to be shifted to one side to prepare the other leg to become a swing leg. In this weight shift motion, the control method that is mentioned in the section II-B is used where the reference CoP trajectory is tracked in a feed back control fashion. The modification of the body motion emerges as a result of this CoP control. The robot executes a weight shift motion by tilting the whole body so as to cope with a constant external force. Note that the CoP trajectory is adequately designed in advance so that the weight shift motion can be achieved.

B. Switching from double support phase

The control in the double support phase is based on the trajectory tracking control of the CoP position. Thus, when the CoP position become greater than a threshold value, the control law is switched from the one for the double support phase to that for single support phase by regarding that the weight shift motion is sufficiently achieved.

C. Control for single support phase

The static balance control law (10) is adopted to the ankle joint in the single support leg. However, the body as well as the swing leg should be moved according to the reference trajectories of stepping motion. The disturbance to the balance caused by these movements will be compensated by the control law.

To generate the body and swing leg movements for stepping motion, the reference trajectories must be provided and the feedback control to them is required for the two hip joints. In order to make the stepping motion adaptive to the environment, these reference trajectories must be modified with the environment, e.g., irregularity of the ground. Here, the reference trajectories are set at the start of the single support phase based on the posture at that moment. Each hip joint should be made extension/flexion in the predetermined amount from the initial angle, and then return to the initial angle. By this method, the trajectory changes adaptively with the environmental conditions.

D. Switching from single support phase

When the foot of the swing leg is placed on the ground, the control law is switched back to that for the double support phase.

E. Simulation

The control method in the previous section is simulated under the influence of the constant external force expressed as the following equations.

$$F_x = -Mg \sin\alpha \tag{18}$$

$$F_y = -Mg(1 - \cos\alpha). \tag{19}$$

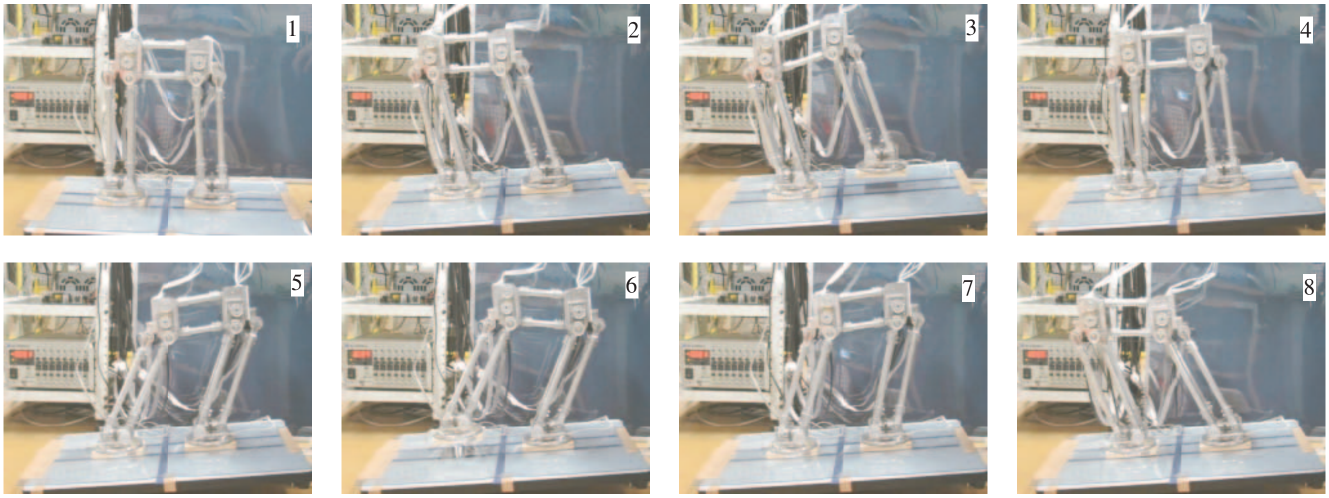


Fig. 5. In-place stepping motion experiment of the slope.

These external forces are equivalent to a set of external forces acting on the biped robot on the slope with slope angle α. The cases where $\alpha = 0$[rad] (no external force) and $\alpha = 0.2$[rad] are examined. Parameters are: $M = 2.5$[kg], $m = 1.25$[kg], $m_f = 0$[kg], $L = 0.20$[m], $\ell = 0.1$[m], $\ell_B = 0.07$[m], $\ell_f = 0.02$[m]. The feedback gains of (6) are set to $K_d = 30$, $K_p = 500$, and $K_f = 1$, while those of (17) are $K_d = 5$, $K_p = 10$ and $K_f = 100$. In the single support phase, the conventional PD control with nonlinear compensation is applied to the hip joints. These feedback gains are set to $K_d = 500$ and $K_p = 100$.

The graphs in Fig. 4(a) and (b) represent the time based plot of the CoP position. Regardless of the external forces, the similar CoP profile are obtained, implying that the weight shift is achieved as intended in both cases. The time based plot of the sway angle ϕ is denoted in 4(c). It can be seen that when the external force is exerted, the stepping motion is performed such that the biped robot tilts the whole body against it.

IV. Experiment

A. Apparatus

The design of the reduced degree of freedom biped robot used in our experiment is originally proposed by Yoneda et al. [10]. This robot has parallel link structure in the leg parts as well as the body part, which keeps the soles parallel to each other. The biped robot is about 40[cm] in height and is about 2.9[kg] in weight. The sole is 12[cm] in length and the horizontal distance between right to left ankle is 20[cm]. Four motors are installed. One of them actuates the motion around yawing axis at ankle joints, which is not used in this experiment. The other three motors are used to achieve a stepping motion within the lateral plane.

A personal computer with the RT-LINUX operating system is used to compute the torque output signal for all motors. The computed output signal is sent via D/A converter to the motor and command the motor to output the desired torque. A motion of the biped robot is detected by the rotary encoder installed in each motor. Three load cells are attached on each sole, from which the position of the CoP is calculated. The controller's sampling time is 1[ms] in this experiment.

B. Control law

The control law mentioned in the section III was adopted. However, in the preliminary experiment, the biped robot tumbled in the single support phase. Thus, the control for ankle joint in the single support phase was changed from eq. (6) to the position control to the reference trajectory. The reference trajectory was set as follows. In the first 3[s], the ankle joint was extended 0.2[rad] from its initial value. The ankle kept the current angle during next 2[s] and then flexed 0.2[rad] to return to the initial angle in the next 3[s]. For the hip joints in the single support phase, on the other hand, the reference trajectories were set as follows. The hip joint of the support leg was extended 0.9[rad] in the first 5[s] and flexed 0.9[rad] in the next 5[s]. The reference trajectory of hip joint of the swing leg was generated based on the state of the support leg so that the two legs become almost parallel.

When the swing leg contacted with the ground, the control law was switched to that for the double support phase. Here, to ensure the ground contacts, the posture at the moment of the contact was kept for 1[s] before starting the control for the double support phase.

The control in the double support phase is the tracking control of the CoP position. The reference trajectory was set so that the CoP moved from the current position to 50mm away from the midpoint of the two ankle joints in 5[s]. The threshold value for switching to the one for single support phase was selected from the experiment and set at 38[mm] away from the mid point of the two ankle joints.

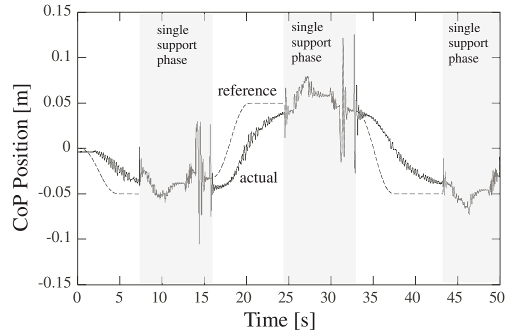
C. Results

The lateral stepping experiments were executed in two conditions: on the flat ground and on 0.1 [rad] slope. As shown in (18) and (19), the slope provides the similar effect of the disturbance by constant external force. The snapshots of the robot motion on the slope are shown in Fig. 5. The time based plot of the CoP position on the flat ground is shown in Fig. 6(a), while the one on the slope is shown in Fig. 6(b). Note here that, in the experiment, the CoP is not controlled in the shaded single support phase. Both CoP profiles of the two experimental conditions are not much different, implying that the stepping motion can be achieved regardless of the slope angle. The time based plot of the sway angle ϕ is shown in Fig. 6(c). The profile of the slope condition is shifted up about 0.1 [rad] from that of the flat condition. This means that the biped robot realizes the stepping motion on the slope by tilting the whole body in the same amount as the slope angle. These results are consistent with the simulation results as shown in Fig.4

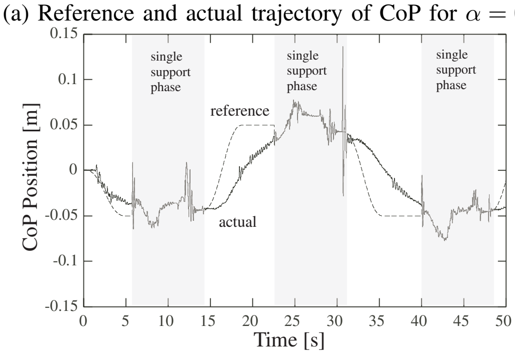
In the actual robot experiment, the control for ankle joint in the single support phase had to be changed from CoP feedback control to the conventional position feedback. One possible reason is the slow response of the control law (10) resulting in the inability to keep the balance with respect to the fast disturbances. Although the simulation results in section III-E show that the response can be made faster with larger feedback control gains, in the actual biped robot, large control gains induced mechanical vibration and the vibration impedes the control states to reach equilibrium points. The other significant reason is the mechanical problem. The slip due to wear at the set screw of the pulley is observed in the mechanical drive system of the hip joints. The slip led to control mismatch and, ultimately, loss of balance. We will fix the mechanical problem and re-examine the effectiveness of the original control law with CoP feedback in the single support phase. The improved in-place lateral stepping performance will be presented in the next paper.

V. Conclusion

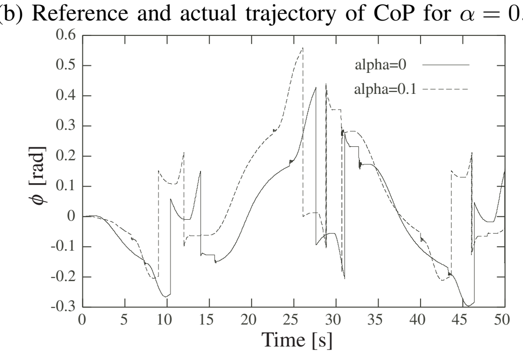
In this paper, we propose an in-place stepping motion control in which the stepping motion is automatically adjusted according to the slope change without modifying the reference trajectory of the CoP. The invariance nature of the reference trajectory with respect to the environmental conditions helps us construct a control law based on the CoP feedback. This control method is basically an extension of the two methods, static balance control and weight shift movement that was proposed in the previous papers. The effectiveness of the control law is confirmed by not only simulation results but also the empirical results. Consequently, the in-place lateral stepping motion is achieved on the slope as well as the flat ground without any modifications of the control law.



(a) Reference and actual trajectory of CoP for $\alpha = 0$



(b) Reference and actual trajectory of CoP for $\alpha = 0.1$



(c) Horizontal position of CoM.

Fig. 6. Experimental results.

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