On an Adaptation in Distributed System Based on a Gradient Dynamics

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ABSTRACT

Distributed system is usually constructed from many homogeneous subsystems with their local couplings, and dose not have a control center. From the structural similarity to biological systems, the distributed system is expected to possess the flexibility or adaptability. To construct a distributed system, two problems arise: One is how to control the whole system by only use local interactions based on local couplings, and another is how to design the adaptation mechanisms. To the former problem, a method based on the gradient system has been discussed in our previous paper. In this paper, we develop this method further and propose an new adaptation rule executable in the subsystems with local couplings. The essence of the adaptation is to adjust the desired system states so that interactions among subsystems are made small. We show a simulation result of a coupled oscillator system where the oscillation pattern changes adaptively with environmental conditions.

1. INTRODUCTION

Autonomous systems can be found in many natural real systems and applications: In a pattern generator in animal locomotion, several neural oscillators are locally coupled in the spinal cord and their local interactions produces a suitable rhythms necessary to walk, swim, or fly. We have proposed a mathematical model of locomotion pattern generator and described an adaptive behavior of decerebrate cats observed on the treadmill[1].

In this paper, developing our model, we consider a more general problem on a control and adaptation of a distributed system under the following framework:

- The whole system consists of m homogeneous subsystems $\{S_i\}(i = 1, \cdots, m)$. The homogeneousness means that the dynamics of subsystems are described by a differential equation of the same form. For simplicity, the state of homogeneous subsystems is expressed by one dimensional state variable q_i. Here, put a vector Q as $Q = [q_1, \ldots, q_m]^T$.
- In distributed systems, there are no one-to-all couplings, in other words, the dynamics of subsystem only contain a state variable of the neighboring subsystems. To discuss the locality of coupling, denote the neighbors of S_i by $N(q_i)$:

$$N(q_i) = \{q_\ell \,|\, q_\ell \text{ has coupling with } q_i\} \qquad (1)$$

It is assumed that q_i is included in $N(q_i)$.

- Between all two subsystems with coupling, e.g., S_i and S_j, a linear functional relation is defined as $p_k = p_k(q_i, q_j)$, where $k(k = 1, \cdots, n)$ is the numbering of couplings. An example of such functional relations is the difference of two subsystem states. When defining $P = [p_1, \cdots, p_n]$, the functional relations are described as

$$P = LQ \qquad (2)$$

where L is a matrix in $R^{n\times m}$.

- A control problem is how to define the subsystem dynamics,

$$\dot{q}_i = f_i(N(q_i)). \qquad (3)$$

so that the functional relations P defined by (2) converge to their desired values $P_d = [p_{d1}, \cdots, p_{dn}]$. Here, it is assumed that the desired relation P_d are set to be consistent, i.e., there exists such Q that satisfies an algebraic equation $LQ = P_d$. This condition can be expressed as

$$(I - LL^\dagger)P_d = 0. \qquad (4)$$

Here, $L^\dagger$ is a pseudo-inverse of L.

- When P converges to P_d, order will be formed over the whole system from a macroscopic view. Here, we call such order as "pattern". The desired pattern P_d, however, is not always consistent under the variations of environmental conditions. In such cases, how to make change of the desired pattern should be considered as an adaptation problem.

2. PATTERN CONTROL BASED ON GRADIENT SYSTEM

Subsystem dynamics for gradient system

H. Yuasa and M. Ito have solved the above problem based on a gradient dynamics[2]. Using vector form, (3) can be generally written as

$$\dot{Q} = F(Q). \qquad (5)$$

Here, $F = [f_1, \cdots, f_m]^T$. Using (2) and (5), the dynamics of P are written as

$$\dot{P} = L\dot{Q} = LF(Q) \qquad (6)$$

They firstly presented conditions such that (6) are described as not only an autonomous system but also a gradient system.

Lemma 1 (H. Yuasa and M. Ito[2]) *The Dynamics of P, i.e., (6), is described as an autonomous system, if and only if F in (5) satisfies*

$$L\frac{\partial F}{\partial Q}(I - L^{\dagger}L) = 0. \tag{7}$$

Lemma 2 (H. Yuasa and M. Ito[2]) *If the dynamics of P is described as a gradient system with a potential function $V(P)$,*

$$P = -\frac{\partial V}{\partial P}, \tag{8}$$

then F can be written as

$$F = \left(\frac{\partial V_X(X)}{\partial X}\right)^T + (I - L^{\dagger}L)Q', \tag{9}$$

where $X = -L^T P$, $V_X(X) = V_X(-L^T P) = V(P)$, and Q' is an arbitrary vector having the same dimension as Q. Conversely, if F is described as (9), then there exist a potential function $V(P)$ using which the dynamics of P is described as (8).

Considering the locality of subsystem coupling, they demonstrated the following theorem.

Theorem 1 (H. Yuasa and M. Ito[2]) *For F in the right hand side of (5), consider the orthogonal decomposition*

$$F = \tilde{F} + \bar{F}, \tag{10}$$

where $\tilde{F} = (I - L^{\dagger}L)F = [\tilde{f}_1, \cdots, \tilde{f}_m]^T \in kerL$ and $\bar{F} = L^{\dagger}LF = [\bar{f}_1, \cdots, \bar{f}_m]^T \in (kerL)^{\perp}$. If each element of $\bar{F}$ is described as a function of x_i, i.e.,

$$\bar{f}_i = \bar{f}_i(x_i), \tag{11}$$

then F in (10) satisfies (9) and the dynamics of P can be described as a gradient system like (8). Here, x_i is the i-th element of vector $X = -L^T P = -L^T LQ$. Moreover,

$$V(X) = \sum_{i=1}^{m} \int \bar{f}_i(x_i)dx_i \tag{12}$$

can be a potential function in (8).

Let us show that the dynamics of q_i is described using only the variables in $N(q_i)$. From the definition of X,

$$x_i = -\sum_{k=1}^{n}\sum_{\ell=1}^{m} L_{ki}L_{k\ell}q_{\ell} \tag{13}$$

where L_{ij} denote the element of the matrix L in the i-th row and j-th column. Now consider the structure of matrix L. Each row of L contains only two non-zero elements corresponding to the coupling subsystems. It follows that, for any given k, L_{kj} and $L_{k\ell}$ is nonzero if and only if subsystem ℓ has coupling with subsystem j. Thus, according to (1), x_i is described only with the state variables in $N(q_i)$.

Design of subsystem dynamics

In order to describe the dynamics of P as a gradient system, we define subsystem dynamics as

$$\dot{q}_i = \tilde{f}_i + \bar{f}_i(x_i). \tag{14}$$

where $[\tilde{f}_1, \cdots, \tilde{f}_m]^T = \tilde{F} \in kerL$ and $[\bar{f}_1, \cdots, \bar{f}_m]^T = \bar{F} \in (kerL)^{\perp}$. The dynamics of P are decided only by $\bar{f}_i(x_i)$ because $L\tilde{F} = 0$. So, in this section, we discuss the conditions for the class of function $\bar{f}_i(x_i)$ so that P converges to its desired value $P_d = [p_{d1}, \cdots, p_{dn}]$.

It can be easily shown that, the dynamics of P reaches its stationary state $\bar{P}$, if and only if $\bar{F}(\bar{X}) = [\bar{f}_1(\bar{x}_1), \cdots, \bar{f}_m(\bar{x}_m)]^T = [0, \cdots, 0]^T$. Here $^-$ denotes a stationary state of dynamic variables. Accordingly, we design $\bar{F}(x)$ so that $\bar{F}(\bar{X}) = \bar{F}(-L^T\bar{P}) = 0$ when the desired pattern emerges, i.e., $\bar{P} = P_d$. Firstly, we discuss the relation between $\bar{P}$ and $\bar{X}$.

Lemma 3 *Suppose P_d is consistent. When X goes to the desired value X_d, P also converges to P_d.*

Proof 1 *By the definition, $X = -L^T P$, and $X_d = -L^T P_d$. Subtracting them, we obtain*

$$X - X_d = -L^T(P - P_d). \tag{15}$$

We should show that, if $\bar{X} - X_d = 0$, i.e., $L^T(\bar{P} - P_d) = 0$, then $\bar{P} - P_d = 0$. From the characteristics of pseudo-inverse,

$$LL^{\dagger} = (LL^{\dagger})^T = (L^{\dagger})^T(L^T) = (L^T)^{\dagger}(L^T). \tag{16}$$

Since P_d satisfies (4), we have

$$\begin{aligned}\bar{P} - P_d &= \bar{P} - LL^{\dagger}P_d = \bar{P} - (L^T)^{\dagger}(L^T)P_d \\ &= \bar{P} - (L^T)^{\dagger}(L^T)\bar{P} = \bar{P} - LL^{\dagger}\bar{P} = [I - LL^{\dagger}]\bar{P} \\ &= [I - LL^{\dagger}]L\bar{Q} = 0 \end{aligned} \tag{17}$$

The above lemma mentions that we can design the dynamics of P in the space of X. Next, we consider the convergence of X.

Lemma 4 *Assume that the subsystems' dynamics are given by (14). If $\bar{f}_i(x_i)$ satisfies the following two conditions:*
(i) $\bar{f}_i(x_i) = 0$ at $x_i = x_{di}$,
(ii) $\left.\frac{\partial \bar{f}_i(x)}{\partial x_i}\right|_{x_i = x_{di}} > 0$.
then $X = X_d$ becomes one of the stationary states, Moreover, if
(iii) $\bar{f}_i(x_i) \cdot (x_i - x_{di}) > 0$,
$X = X_d$ becomes the unique stationary state.

Proof 2 *When subsystem dynamics are given by (14), the dynamics of P becomes a gradient system, the potential function of which is given by (12). Thus, we should show that $P = P_d$, i.e., $X = X_d$ corresponds to a local minimum point of potential function V in (12). The necessary and sufficient conditions for this are that $\frac{\partial V}{\partial p_i} = 0$ and all the eigen values of Hessian matrix H_V of V are positive at $X = X_d$. Executing some calculations, we can obtain*

$$\frac{\partial V}{\partial P} = -L\bar{F}(X), \tag{18}$$

$$H_V = LD_{\bar{f}}L^T, \tag{19}$$

where $D_{\bar{f}} = diag[\frac{\partial \bar{f}_1(x_1)}{\partial x_1}, \cdots, \frac{\partial \bar{f}_m(x_m)}{\partial x_m}]$. It follows from these equations that, if both conditions (i) and (ii) hold, $X = X_d$ is a local minimum point of potential function. Moreover, if condition (iii) are satisfied, there are no points that satisfies (i) and (ii) except $X = X_d$. Thus, $X = X_d$ is a global minimum point of potential function.

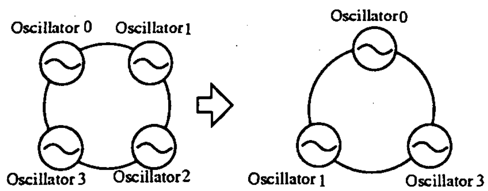


Figure 1: A circularly-coupled oscillator system and subsystems' breakdown.

Note 1 *Using a monotonous increasing odd function f_i^+, one of $\bar{f}_i$'s satisfying conditions (i)-(iii) is given by,*

$$\bar{f}_i = f_i^+(x_i - x_{di}). \tag{20}$$

We can summarize the above lemmas as follows.

Theorem 2 *Assume that P_d satisfies (4) and a function $\bar{f}_i(i = 1, \cdots, m)$ satisfies three condition (i) - (iii) in lemma 4. Then, for the system whose subsystem dynamics are constructed as (14), $P = LQ$ converges to P_d.*

An example: Relative phase control in coupled oscillator system

For an example of distributed control system, we take here relative phase control in a circularly-coupled oscillator system. H. Yuasa and M. Ito also applied their theory to a coupled oscillator system, and modeled a central pattern generator (CPG) in animal locomotion [3]. This problem is very compatible to the framework considering in this paper: subsystems correspond to oscillators, and difference in phases between coupled oscillators is described as a linear relation. Then, matrix L is expressed as incidence matrix of the graph, where vertices and edges respectively correspond to oscillators and their couplings.

For example, consider the coupled 4-oscillator system, as shown at the left side figure in **Fig. 1** . The goal here is to control the relative phase between two coupled oscillator i and $i+1$: let $p_i = q_i - q_{i+1} (i = 0, 1, 2, 3)$ converge to their desired value $p_{di} (i = 0, 1, 2, 3)$. Assume $q_4 = q_0$. Then,

$$P = LQ, L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix}, \tag{21}$$

where, $P = [p_0, p_1, p_2, p_3]^T$ and $Q = [q_0, q_1, q_2, q_3]^T$. The kernel space of matrix L is $[1, 1, 1, 1]^T$ and from the relation $X = -L^T LQ$, $X = [x_0, x_1, x_2, x_3]$ becomes,

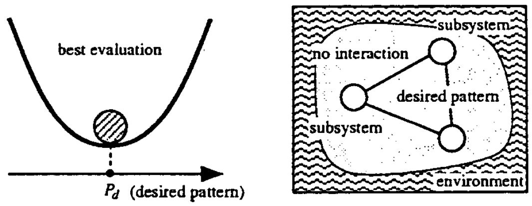
$$x_i = p_{i-1} - p_i = q_{i-1} - 2q_i + q_{i+1}. \tag{22}$$

Here, $p_{-1} = p_3$, $p_4 = p_0$ and $q_{-1} = q_3$. Accordingly, we set the oscillator dynamics as

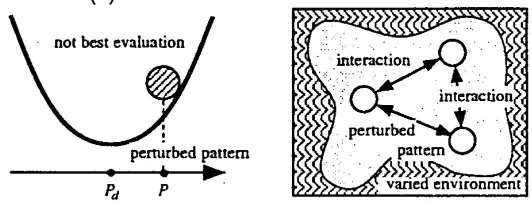
$$\dot{q}_i = \omega + f_i^+(x_i + p_{di} - p_{di-1}) \ (i = 0, 1, 2, 3), \tag{23}$$

where $p_{d-1} = p_{d3}$. On account of the periodicity of oscillators, we select, for f_i^+, a periodic function,

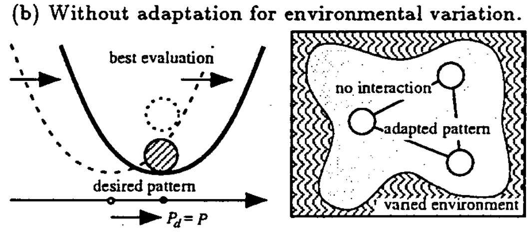
$$f_i^+(x) = \tau_i \sin\frac{x}{4} \tag{24}$$



(a) Before an environment variation.



(b) Without adaptation for environmental variation.



(c) With adaptation for environmental variation.

Figure 2: Our concept on adaptation. The left figure denotes an evaluation of formed-pattern, while the right figure expresses the sketch of subsystem interactions between coupled ones.

instead of a monotonous increasing function. Finally, we obtain

$$\dot{q}_i = \omega + \tau_i \sin\frac{1}{4}(q_{i-1} - 2q_i + q_{i+1} + p_{di} - p_{di-1}). \tag{25}$$

Here, τ_i determines the magnitude of the effect from coupled oscillators. The dynamics of pattern P becomes

$$\begin{aligned} \dot{p}_i \quad = \quad & \tau_i \sin\frac{1}{4}(p_{i-1} - p_i - p_{di-1} + p_{di}) \\ & - \tau_{i+1} \sin\frac{1}{4}(p_i - p_{i+1} - p_{di} + p_{di+1}), \end{aligned} \tag{26}$$

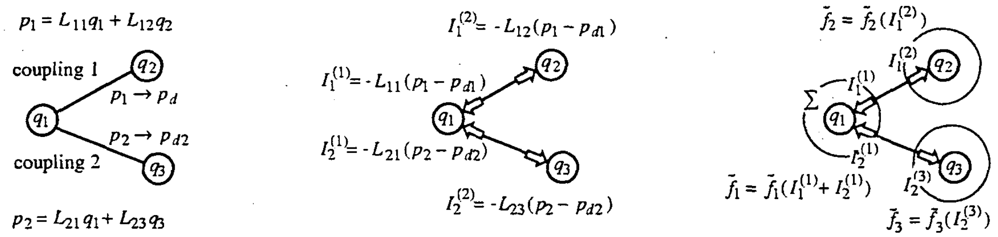
Here, $p_{d4} = p_{d0}$ and $\tau_4 = \tau_0$. These dynamics certainly described as a gradient system of the following potential function V

$$V = -\sum_{i=0}^{3} 4\tau_{i+1} \cos\frac{1}{4}(p_i - p_{i+1} - p_{di} + p_{di+1}). \tag{27}$$

3. ADAPTIVE CHANGES OF PATTERNS

Control and Adaptation

Theorem 2 gives conditions ensuring that P converges to P_d as long as the desired pattern P_d is consistent. However, when the environments vary, the consistency of P_d is sometimes broken. Let us consider a case of a circularly-coupled oscillator system in the previous section. It is often



(a) relative phase p_j. (b) Interactions I_{ij}. (c) Effect from coupled subsystems $\tilde{f}_i$.

Figure 3: Relation and interaction between adjacent subsystems.

required, in a coupled oscillator system, that each of them should work periodically with the same constant relative phases*. Note that, in this requirement, the desired relative phases are influenced from the total number of subsystems: if one of subsystems breaks down † as shown in Fig. 1, the desired phasic relation will change. At that time, the desired pattern P_d before breakdown lose consistence.

When P_d is inconsistent, there is no Q that satisfies $P = LQ = P_d$. Thus, P_d should be adjusted so that it becomes suitable to the varied environment and there exists such Q, i.e., satisfies (4) again. We call such an adjusting function, "adaptability". In this section, we treat this topic within the framework of a gradient system.

In order to make a system evolve towards a more suitable one by adaptation, an evaluation for the current system is necessary. From the view point of engineering, an evaluation should be good if P is close to P_d. Therefore, we define the following evaluation function,

$$E = \frac{1}{2}\sum_{i=1}^{n}(p_i - p_{di})^2. \tag{28}$$

When the desired value P_d is realized, an evaluation becomes best, as shown at the left figure in **Fig. 2** (a). Then, as shown latter, local interactions among subsystems are zero.

Now, suppose that P_d becomes not to satisfy (4) by the changes in environment such as subsystems' breakdown. Then, the formed pattern has shifted from the desired one, as depicted at the left figure in **Fig. 2** (b). Then, interactions between coupled subsystems always work. Such a situation is, however, not so good, because the pattern is formed as the balance of interactions, as the right figure in **Fig. 2** (b). Actually, when P_d was consistent and the desire pattern emerged, these interaction had disappeared as shown in **Fig. 2** (a).

Accordingly, we try to decrease subsystem interactions and cancel their balance with keeping the emerging pattern, as shown in the left figure in **Fig. 2** (c). This scheme is equivalent to adjusting P_d so as to satisfy (4). At this time, the evaluation function (28) also changes so that a new formed pattern obtains the highest evaluation, as depicted in the left figure in **Fig. 2** (c).

To achieve both the convergence to and adjustment of P_d, it is important to split the dynamics by the time scale: the system dynamics and the adaptation dynamics. Since adaptation usually works after evaluating the formed pattern, the adaptation dynamics should be enough slower than the system dynamics. The difference in time scale is essential in an adaptive systems.

Subsystem Interactions

Before discussing the adaptation dynamics, we should clarify how subsystem interactions are represented in subsystem dynamics. From Theorem 2 and Note 1, we set subsystem dynamics as

$$\dot{q}_i = \tilde{f}_i + f_i^+(x_i - x_{di}). \tag{29}$$

The effect from the coupled subsystems are expressed in the term f_i^+. Thus, we examine the description of $x_i - x_{di}$ to define the interactions. According to (15),

$$x_i - x_{di} = -\sum_{k=1}^{n} L_{ki}(p_k - p_{dk}). \tag{30}$$

Here, $L_{ki} \neq 0$ if and only if the coupling numbered by k couples subsystem S_i with another. At this time, the influence the subsystem S_i gets is in proportion to the difference between a functional relation and its desired value defined at the coupling k. Thus we define the subsystem interaction $I_k^{(i)}$ as

$$I_k^{(i)} = -L_{ki}(p_k - p_{dk}), \tag{31}$$

which denotes the influence to the subsystem i through the k-th coupling, as shown in **Fig. 3** (b). By this definition, the second term of (29) becomes

$$f_i^+(x_i - x_{di}) = f_i^+(\sum_{k=1}^{n} I_k^{(i)}). \tag{32}$$

This equation indicates that, in order to actually give effect to the dynamics of subsystem S_i, $I_k^{(i)}$ have to be locally summed up and then mapped by f_i^+ ‡.

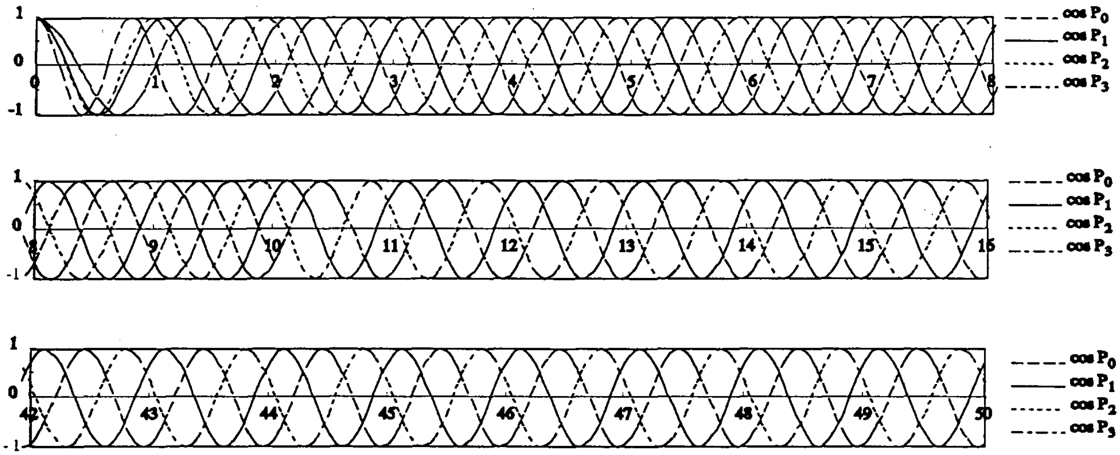
Using (31), the evaluation function (28) can be written as

$$E = \frac{1}{4}\sum_{i=1}^{m}\sum_{\substack{k=1 \\ L_{ki}\neq 0}}^{n} \frac{1}{L_{ki}^2}\{I_k^{(i)}\}^2. \tag{33}$$

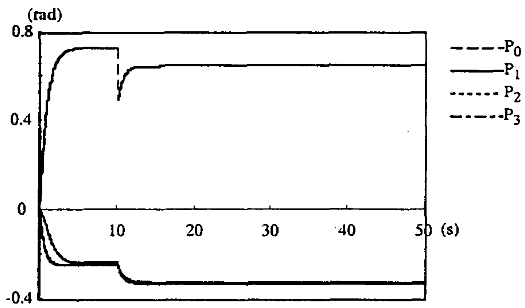
*Such an example is found in a multi-cylinder engine, where each cylinder (or its valve) correspond to subsystem. In one cycle, each valve is controlled by a cam-shaft at the same constant relative phases.

†At this example, the reliability of subsystems is regarded as environmental condition.

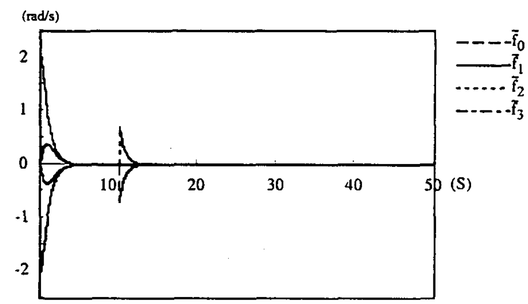
‡Such a mapping can be found in the sigmoid function in neuron model. It is the same in that it maps the weighted sum of the direct action from the connected neurons.



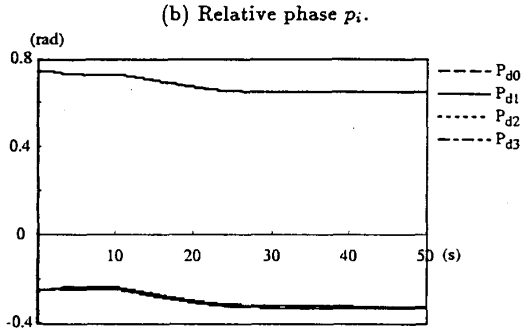
(a) Oscillator phase θ_i.



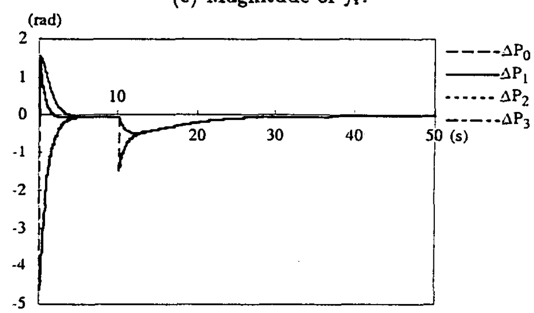
(b) Relative phase p_i.



(c) Magnitude of $\bar{f}_i$.



(d) Desired relative phases P_{di}.



(e) Difference between P and P_d.

Figure 4: Simulation results when the number of subsystems decreases in the coupled oscillator system.

This means that the pattern can be also evaluated by the magnitude of the interactions.

Adaptation dynamics

This section presents the adjusting method of P_d. Note that, even if P_d have changed, the dynamics of P is still described by a gradient dynamics.

For consistent P_d, $\bar{X} - X_d = 0$ if and only if $\bar{P} - P_d = 0$. Then, not only $\bar{F}(\bar{X} - X_d) = 0$ but also $\bar{I}_k^{(i)} = 0$ are satisfied. However, if P_d becomes inconsistent and unsatisfying (4) by environmental variations, then $\bar{P} - P_d \neq 0$, nonetheless $\bar{X} - X_d = 0$. This means, from (15), that $\bar{P} - P_d \in ker L^T$. Therefore, $F^+(\bar{X} - X_d) = 0$, but $\bar{I}_k^{(i)} \neq 0$, implying that the emerging pattern is maintained on the balance of interactions among subsystems. Our goal is to decrease interaction $\bar{I}_k^{(i)}$ to prevent such a situation. This is achieved by adjusting P_d so that the emerging pattern coincides with the desired one.

Lemma 5 *Define the adaptation dynamics of P_d as*

$$\dot{p}_{di} = \tau_p(p_i - p_{di}), \tag{34}$$

where $0 < \tau_p \ll 1$. *Then, these dynamics do not increase* $|f_i^+|$.

Proof 3 *When* τ_p *is small enough, the pattern dynamics of* P *is regarded as much faster than the adaptation dynamics (34). Accordingly, at the time scale of the dynamics of* P_d*, the dynamics of* P *have already converged to the stationary state* $\bar{P}$. *Then,* x_i *is also stationary, i.e.,* $x_i = \bar{x}_i$. *Here, let* $y_i = \bar{x}_i - x_{di} = -\sum_{j=1}^{n} L_{ji}(\bar{p}_j - p_{dj})$*, then, it follows from* $\dot{y}_i = -\tau_p(\bar{x}_i - x_{di})$ *that*

$$\begin{aligned} \dot{f}_i^+(\bar{x}_i - x_{di}) &= \dot{f}_i^+(y_i) = \frac{\partial f_i^+(y_i)}{\partial y_i}\dot{y}_i \\ &= -\tau_p \frac{\partial f_i^+(\bar{x}_i - x_{di})}{\partial \bar{x}_i}(\bar{x}_i - x_{di}). \quad (35) \end{aligned}$$

From Note 1, f^+ *is a monotonous increasing function, i.e.,* $\frac{\partial f_i^+(\bar{x}_i - x_{di})}{\partial \bar{x}_i} > 0$. *Therefore we obtain* $\frac{d}{dt}|\bar{f}_i^+| < 0$ *for* $\bar{x}_i \neq x_{di}$. *The equality* $\frac{d}{dt}|\bar{f}_i^+| = 0$ *holds only at* $\bar{x}_i = x_{di}$.

Note 2 *Applying an adiabatic approximation of synergetics [4], the dynamics of* P *can be regarded as stationary in the dynamical scale of* P_d. *In this dynamical scale, not only* $\bar{f}_i^+ = 0$*, but also* $\bar{X} = X_d$ *from lemma 4, are always satisfied. These equations together with (15) indicate that* P_d *evolves so that* $\bar{P} - P_d$ *stays within the kernel space of matrix* L.

We verify the following lemma on the convergency of P_d.

Lemma 6 *Suppose that* τ_p *is small enough for satisfying an adiabatic approximation. Let* P_d *evolve in time according to (34). Then,* P_d *coincides with* $\bar{P}$.

Proof 4 *From an adiabatic approximation, we can regard* p_i *as constant value* $\bar{p}_i$. *Select (28) for Lyapunov function. Then* E *in (28) satisfies* $E \geq 0$*, and*

$$\dot{E} = -\sum_{i=1}^{n} \tau_p(\bar{p}_i - p_{di})^2 \leq 0. \quad (36)$$

The equality holds at $p_{di} = \bar{p}_i$. *Therefore,* $p_{di} = \bar{p}_i$ *is stable and thus* P_d *converges to* $\bar{P}$.

Note 3 *The* P_d *obtained from the adaptation dynamics (34) is consistent, i.e., it satisfies (4).*

We can summarize the above lemmas as follows:

Theorem 3 *For the system given by (29), define the adaptation dynamics as (34). If the time scale of (34) is enoughly smaller than that of (29), then* P *and* P_d *are equal at the stationary state. At this time,* P_d *satisfies (4).*

Proof 5 *The former of this lemma has been proved by the above lemmas. On the other hand, the latter is also trivial from the proof of lemma 3.*

4. SIMULATION

We executed the simulation for a circularly-coupled 4-oscillator system shown in Fig. 1. We set the desired oscillation pattern as a pattern such that each oscillators oscillate at the same constant relative phases in no relation to the total number of oscillators. Since the desired oscillation pattern depends on the total number, the oscillator 2 was removed at 10 seconds from the start of simulation, which correspond to the environmental changes. All the subsystems never know when and which subsystems are removed.

Fig. 4 (a) shows the time evolution of the phase for each oscillators. Although all the phases are the same at the initial state, oscillators create the desired oscillation pattern with the same relative phases in some seconds. However, this oscillation pattern is perturbed at 10 second owing to the change in the total number of oscillators. Nevertheless, the new oscillation pattern emerges after a while according to oscillator interactions. The new oscillation pattern is maintained until the end of the simulation.

Fig. 4 (b) shows the evolution of the relative phases p_i, while **Fig. 4** (c) depicts the magnitude of f_i^+. When $f_i^+ = 0$, the system stays at a stationary state and relative phases keep constant. They converged to $-\frac{1}{2}\pi$ (or $\frac{3}{2}\pi$) in 4-oscillator system, while $-\frac{2}{3}\pi$ (or $\frac{4}{3}\pi$) in 3-oscillator system. These results indicate that the suitable oscillation pattern is obtained according to the total number of oscillators.

Fig. 4 (d) depicts the changes of p_{di} and **Fig. 4** (e) shows $p_i - p_{di}$ that is proportional to the oscillator interaction $I_k^{(i)}$. Just after the change of environment, oscillator interactions are not eliminated in spite of $f_i^+ = 0$. This indicates that the oscillation pattern is maintained by the balance among oscillator interactions. By an adjustment of p_{di}, however, the differences between p_i and p_{di} vanish and interactions disappear. Finally, the proper desired relative phases are obtained in p_{di}.

5. CONCLUSION

We considered an adaptation in the distributed system on the basis of control theory following a gradient dynamics proposed by H. Yuasa and M. Ito. We define the adaptation dynamics, whereby the desired pattern is adjusted with slow dynamics so that interactions among subsystems decrease. We executed the computer simulation for coupled-oscillator system, which shows that a suitable oscillation pattern are achieved as well as desired relative phases are properly acquired through the interactions among subsystems. As future works, we consider applications in designing flexible artificial systems.

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