A Mathematical Model of Adaptation in Rhythmic Motion to Environmental Changes

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ABSTRACT

This paper studies the adaptability of the animals' rhythmic movements to the changes in their environments. Rhythmic movements are usually modeled by central pattern generator (CPG) using coupled oscillators. In our research, by analyzing the perturbed locomotion of the decerebrate cat, we propose a mathematical model for an adaptation mechanism. In this model, we take into account the environmental changes and give the adaptation rules so as to minimize the interaction between each oscillators. We also propose a general framework describing adaptive behaviors in rhythmic motion.

1. INTRODUCTION

Many movements of animals, such as locomotion, breathing and chewing, can be executed even though there is no motor command from higher nervous system. These movements are automatic and usually forms periodic patterns. Physiological experiments demonstrate that rhythmic movements are produced in the spinal neuronal network called CPG (Central Pattern Generator), which include neural oscillators [1], and many researchers have formulated the CPG as coupled oscillators.

Recent biological studies further discovered that, during performing the rhythmic movement, an animal has an ability to adapt to its environmental changes. For example, Yanagihara et al. studied the motor learning ability of a decerebrate cat walking on a specially designed treadmill [2]. The treadmill consists of three moving belts. As shown in Fig. 1, in the experiment, they put the left forelimb (LF) and the left hindlimb (LH) of the cat on two independent belts, and put the right two limbs (forelimb (RF) and right hindlimb (RH)) together on the remaining belt. They first drove the three treadmill belts with the same low speed (36cm/s) and made the decerebrate cat to walk with a normal gait pattern called by "walk" (Fig. 2 (a)). After that, they increased the speed of the LF belt to 1.7 times faster (61cm/s) than that of the others (36cm/s). In this case, whenever a cat places its left forelimb onto the belt, the limb suffers from a mechanical perturbation. They then found that the cat's locomotion is initially not stable, that is, the cycle period of two forelimbs often differ in each step cycle. However, after a few training, the cat gradually adapts to a new environment and begins to walk with a new steady gait pattern. The experimental results of gait diagrams are shown in Fig. 2. In this figure, there are two bisupport phases in one step cycle, marked by B_1 and B_2. In B_1 bisupport phase, the left forelimb first support and then comes the right forelimb. Conversely, in B_2 bisupport phase, the right forelimb first support and then the left forelimb. It is found that, during the normal locomotion, the two bisupport phases have equal duration (Fig. 2 (a)). However, perturbation makes B_1 shorter than B_2 (Fig. 2 (b)). This difference tends to decrease when the cat adapts to a new environmental conditions (Fig. 2 (c)). It indicates that the interlimb coordination, which has been disturbed by perturbation, is regained after many steps on treadmill. This interlimb coordination is important to execute the smooth and stable locomotion. Furthermore, it was found that, at the next trial after rest, the cat can walk under the perturbed environment with the gait Fig. 2 (c) from just the beginning of locomotion, which means that the decerebrate cat can memorize the gait pattern that have acquired in adaptation. From the biological point of view, Yanagihara et al. suggest that synaptic plasticity in cerebellum is essential for such adaptation [3].

The object of this paper is to propose a mathematical framework to model the adaptive mechanism of rhythmic movement to environmental changes. In this paper, we first model gait pattern generation as the coordination among each oscillator. We describe the dynamics of relative phases between oscillators as a gradient system. The minimum of the potential function corresponds to the stable gait pattern. According to Yuasa and Ito [4], we can uniquely design the interaction for each oscillator. Then, by taking into account the perturbation from environment, we propose an adaptation rule to adjust the oscillator dynamics as well as the potential function so as to decrease the interaction between the oscillators.

2. GAIT PATTERN GENERATION

Model of rhythmic movement

As shown in Fig. 3, we model the rhythmic movement of locomotion as the coupled oscillators. Yuasa and Ito [4] showed that, if we design the dynamics of relative phase as a gradient system, then we can uniquely design the interaction between each oscillators. A gradient system is a system whose dynamic property can be described using a potential function. The minimum of the potential function corresponds to the stable state of the gradient system. Followed by Yuasa and Ito [4], we use four oscillators to describe the movement of each limb of quadrupeds. We design the connections among each oscillator as shown in

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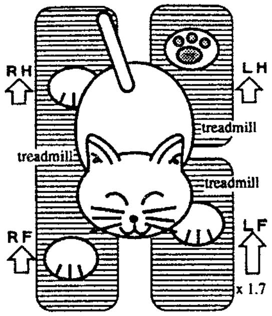
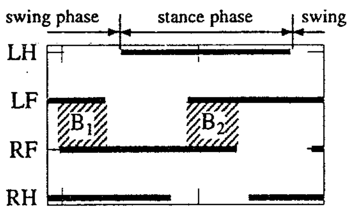
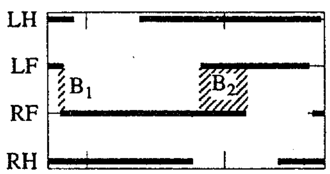


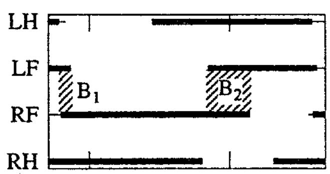
Figure 1: Perturbed locomotion with a decerebrate cat.



(a) Normal locomotion



(b) Before adaptation



(c) After adaptation

Figure 2: Gait diagrams of cat locomotion. They change by adaptation as well as perturbation.

Fig. 3. Note that, this kind of connections is used only for mathematical convenience and may not always coincide with the actual CPG connections in animals. However, it does not influence the essence of the problem.

In Fig. 4, the oscillator phase $\theta_i (i = 0, 1, 2, 3)$ represents a limb state in one cycle of movement. It implies that oscillator phase coincides with the phase of limb movement. Each limb takes two states: swing phase and stance phase. We divide the phase space of oscillators $[0, 2\pi)$ into two parts and assign them to swing phase and stance phase, respectively. As shown in Fig. 4, the range of θ_i where $\cos\theta_i \geq \gamma$ is stance phase and the range where $\cos\theta_i < \gamma$ is swing phase. Here, γ is determined from duty factor β, which denotes the ratio of stance phase in one step cycle. The relation between γ and β is described by

$$\gamma = \cos \pi\beta. \quad (1)$$

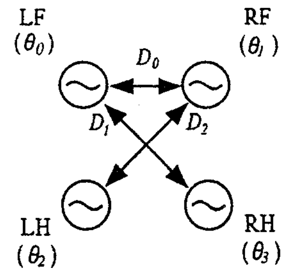


Figure 3: Connection of oscillators in our CPG model (LF: left forelimb, RF: right forelimb, LH: left hindlimb, RH: right hindlimb).

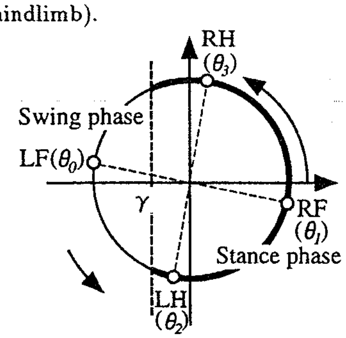


Figure 4: Stance phase and swing phase. In this case, only the LF (θ_0) is in swing phase and the others (LH, RF, RH) are in stance phase.

Dynamics in stance phase: In the stance phase, limbs always contact with the treadmill and can not move freely. Thus, we describe the dynamics of the supporting limbs as follows:

$$\dot{\theta}_i = \rho_i \quad (i = 0, 1, 2, 3) \quad (2)$$

where $\rho_i (i = 0, 1, 2, 3)$ is a variable representing the speed of treadmill belt. Equation (2) means that the limb movement is forced by treadmill.

Dynamics in swing phase: In this phase, limbs can move freely. Thus, it is possible to adjust the phase of limb movement using interaction among each oscillators.

$$\dot{\theta}_i = \omega_i + f_i \quad (i = 0, 1, 2, 3) \quad (3)$$

where $\omega_i (i = 0, 1, 2, 3)$ denotes the angular velocity in swing phase and $f_i (i = 0, 1, 2, 3)$ denotes the interaction term. According to Yuasa and Ito [4], $f_i (i = 0, 1, 2, 3)$ can be given as follows:

$$f_0 = \tau_\theta(\theta_1 + \theta_3 - 2\theta_0 - D_0 - D_1) \quad (4)$$

$$f_1 = \tau_\theta(\theta_0 + \theta_2 - 2\theta_1 + D_0 - D_2) \quad (5)$$

$$f_2 = \tau_\theta(\theta_1 - \theta_2 + D_2) \quad (6)$$

$$f_3 = \tau_\theta(\theta_0 - \theta_3 + D_1) \quad (7)$$

where τ_θ is a constant parameter which controls the magnitude of oscillator interaction, and $D_j (j = 0, 1, 2)$ is a desired relative phase. It should be noted that, if the relative phase equals to the desired value, then the interactions will be zero, i.e., never work.

The desired relative phase $D_j (j = 0, 1, 2)$ is a parameter which describes a particular locomotion pattern, and

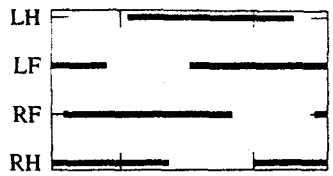


Figure 5: Normal locomotion.

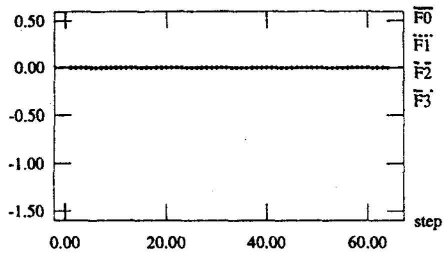


Figure 6: Oscillator interaction during normal locomotion. No interaction works.

$\omega_i (i = 0, 1, 2, 3)$ is another parameter to affect the phase relation among each limbs.

Treadmill dynamics: For a decerebrate cat walking on the treadmill, the rotation velocity of the treadmill corresponds to its environment. We describe it as

$$\rho_i = \mu_i \quad (i = 0, 1, 2, 3), \tag{8}$$

where $\mu_i (i = 0, 1, 2, 3)$ is a parameter describing environment.

Simulation without adaptation

Normal locomotion: In the case when all the treadmill belts are driven with the same speed,

$$\mu_0 = \mu_1 = \mu_2 = \mu_3 = \omega \tag{9}$$

the decerebrate cat can walk with the normal locomotion pattern shown in Fig. 2 (a).

In our model, if we choose

$$\omega_0 = \omega_1 = \omega_2 = \omega_3 = \omega \tag{10}$$

$$D_0 = \frac{1}{2}\pi, D_1 = \frac{3}{4}\pi, D_2 = -\frac{1}{4}\pi, \tag{11}$$

then we can realize the same locomotion pattern as Fig. 2(a). We select the duty factor from Fig. 2(a) ($\beta \cong 2/3$). Thus using eq. (1), we get $\gamma = -0.5$. Fig. 5 shows the simulation result of the gait diagram when the treadmills are driven with the same speed.

In our model, oscillator interaction is regarded as the force that realizes a desired motion pattern. If a desired pattern has already been realized, then interaction equals zero. This is shown in Fig. 6 which plots $F_i (i = 0, 1, 2, 3)$,

$$F_i = \int_T f_i dt \quad (i = 0, 1, 2, 3), \tag{12}$$

that is, the integration of interaction f_i during one step cycle. The potential function with the above minimum

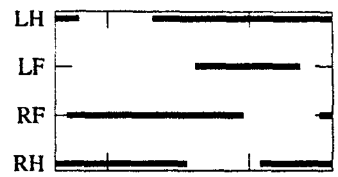


Figure 7: Perturbed locomotion without adaptation.

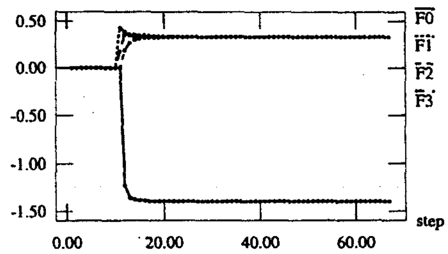


Figure 8: Oscillator interaction in perturbed locomotion. The interaction works so that locomotion should converge to memorized pattern.

(eq. (11)) corresponds to the memorized locomotion pattern (Fig. 2 (a)) in the normal environment.

Perturbed locomotion: When the treadmill belt of the left forelimb (LF) is driven faster (1.7 times) than the others, that is,

$$\mu_0 = 1.7\omega, \mu_1 = \mu_2 = \mu_3 = \omega \tag{13}$$

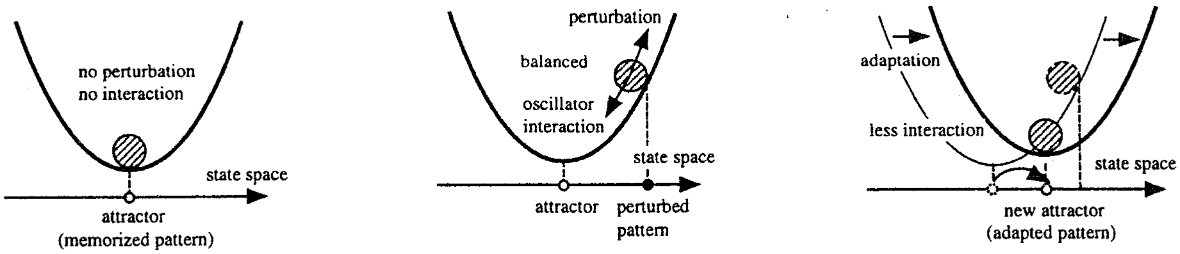
and, if we still use above equation (10) and (11), we then get the simulation result as shown in Fig. 7 and Fig. 8. In Fig. 8, we found that the interactions never goes to zero, which means that there are always interactions between each oscillators.

3. ENVIRONMENTAL ADAPTATION

Adaptation mechanism

In the simulation of previous section (Fig. 7), although we can get stable pattern even though the environment changes, we can not regard this gait pattern as a result of adaptation. In the biological experiments, it was suggested that, firstly, before the cat can walk stably under the perturbed environment, it is necessary to train for a lot of steps; and secondly, after the learning, the cat can memorize the new gait pattern that adapts to the new perturbed environment. However, the mathematical model described in the above section was not sufficient to include these two important aspects.

In fact, as shown in Fig. 9, the minimum point of the potential function corresponds to the point attractor in the relative phase space, and gradient force corresponds to the oscillator interactions. Therefore, the interactions work in such a way that the relative phase converges to the attractor, i.e., a desired state. This ensures the stability of the desired locomotion pattern. The gradient force, i.e., oscillator interaction brings the locomotion from perturbed one to the memorized pattern. When there is a periodic perturbation, the state of relative phase will be



(a) normal locomotion.

(b) perturbed locomotion without adaptation.

(c) perturbed locomotion with adaptation.

Figure 9: Mechanism on adaptation in perturbed locomotion.

shift from the attractor, i.e., minimum point of potential. In this case, the gradient force acts to lead the state back to the attractor. However, since perturbation is periodic, it may, if large, operate before converging to minimum point of potential. As a result, a new state emerges, where perturbation and gradient force are balanced (see Fig. 9(b)). This corresponds to the gait pattern given in Fig. 7. If the new pattern is generated by the balance between the oscillator interaction and periodic perturbation, then the oscillator interaction will always be necessary. This kind of pattern generation is not effective even from the point of view of energy loss. Therefore, we suggest that in the perturbed environment, the potential function itself should be adjusted so that, when performing adaptive movement, the interaction among each oscillator goes toward zero. As shown in Fig. 9(c), it equals to change the minimum point of the potential function.

In summary, our concept about adaptive mechanism is to adjust the parameter of oscillator dynamics, especially potential function, which corresponds to memorized motion pattern. The adjusted parameters in oscillator dynamics determines a emerging motion pattern, whereas the adjustment process corresponds to the learning.

Dynamics of adaptation mechanism: We now study on how to adjust the parameters of locomotion pattern: the angular velocity in swing phase $\omega_i (i = 0,1,2,3)$ and the desired relative phase $D_j (j = 0,1,2)$. A criterion for parameter adjustment is to minimize the oscillator interaction. The adaptation dynamics should be slower than that of locomotion, because it is necessary to firstly know the evaluation of current pattern before adjusting the parameters. The resultant adjustment rule are given as follows:

$$\omega_i^{(n+1)} = \omega_i^{(n)} + \tau_\omega \int_T f_i dt \quad (i = 0,1,2,3), \tag{14}$$

$$D_0^{(n+1)} = D_0^{(n)} + \tau_D \int_T (f_0 - f_1) dt, \tag{15}$$

$$D_1^{(n+1)} = D_1^{(n)} + \tau_D \int_T (f_0 - f_3) dt, \tag{16}$$

$$D_2^{(n+1)} = D_2^{(n)} + \tau_D \int_T (f_1 - f_2) dt, \tag{17}$$

where n denotes a iteration number of step cycle, T is a duration of one step cycle, $f_i (i = 0,1,2,3)$ denotes the force given by eq. (4)–(7) and, τ_ω and τ_D are parameters that influence convergency of ω_i and D_j, respectively. These adaptation dynamics is applied in every step cycle.

Equation (14) controls natural frequency of each oscillator ω_i. We can show that F_i, the integration of interaction f_i for one cycle (eq. (12)), decreases according to eq. (14) (see Appendix). On the other hand, eq. (15) to (17) balance the magnitude of oscillator interactions. Consequently, these equations reduce the following V_D,

$$V_D = \int_T \sum_{i=0}^{3} \left\{ \frac{1}{2\tau_{\theta_i}} f_i^2 \right\} dt, \tag{18}$$

that is, an integration of squared summation of interaction f_i. As a result, the minimization of F_i and V_D reduce the interaction f_i. It should be noted that if the change of ω_i is slow enough, then the dynamics of the relative phase will still keep to be the gradient system.

Simulation: We executed the new simulation by applying the above adaptive mechanism. In the simulation, we set time constants as $\tau_\theta = 2.0$, $\tau_\omega = 0.25$ and $\tau_D = 0.02$.

Fig. 10 shows the gait diagram from simulation, which is much similar to the experimental result of the decerebrate cat in Fig. 2(c). Fig. 11 and Fig. 12 show the time evolution of F_i (eq. (12)) and V_D (eq. (18)), respectively. Fig. 13 and Fig. 14 show the adjustment of angular velocity ω_i and desired relative phase D_j. These figures indicate that parameters of locomotion pattern are changed, which decreased the oscillator interactions.

4. DISCUSSIONS

General features

We have described our concept of adaptive mechanism in rhythmic movements. There are two types of parameters that determine rhythmic motion pattern: parameters representing subsystem's natural characteristics and parameters describing the relation among each subsystems. In the locomotion example, the former is $\omega_i (i = 0,1,2,3)$ and the latter is $D_j (j = 1,2,3)$. If these parameters are adjusted, the attractor of motion dynamics will change, which, in our concept, corresponds to adaptation.

In order to generalize our concept of adaptation, we summarize several general features in the adaptation of rhythmic movement.

Memorization of rhythmic motion patterns: The fact that the same rhythms can be generated repro-

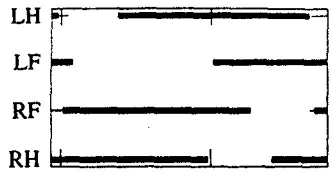


Figure 10: Adapted locomotion to perturbation.

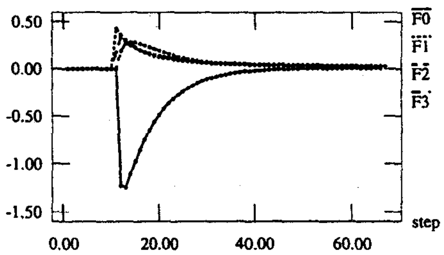


Figure 11: Oscillator interaction in adapted locomotion. The interaction decrease with adaptation.

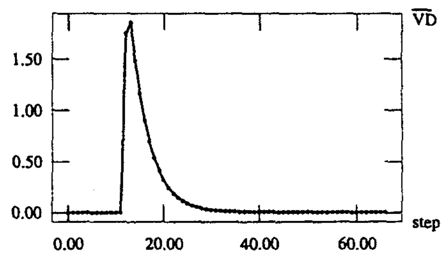


Figure 12: Evaluation function V_D, which decrease with adaptation.

ducibly against the same environment indicates that patterns are memorized in some forms. In our model, natural frequency of oscillator and desired relative phase among oscillators are stored in memory.

Adjustment of rhythmic motion pattern in memory: To adapt to any environments, we need not to memorize all the motion patterns, and it is also impossible against an unexperienced environment. When working in new environment, the corresponding new pattern should be acquired by adjusting the memorized pattern.

Environmental changes: CPG basically produces rhythms in a feedforward manner, since it do not use sensory feedback or command from upper nerve system [1]. If the environment is always changing randomly, such a feedforward rhythm generation will be impossible because environment must be identified by using sensory feedback. Therefore, in order to adjust the CPG so that it generate the adequate rhythmic pattern with respect to the specifical environment, the environment itself should not fluctuate faster than adaptation.

Time scale of adaptation: Adaptation requires the evaluation of present motion. Before pattern adjustment, it is necessary to know how suitable the present pattern is. Therefore, the dynamics of adaptation must be slow enough compared to that of the rhythmic motion dynamics.

Convergency of adaptation : If adaptation progresses slowly, rhythmic motion has to continue for a long time. The repetition of rhythmic motion is important for convergence of adaptation.

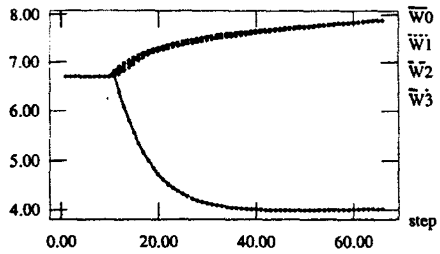


Figure 13: Change of $\omega_i (i = 0, 1, 2, 3)$ by adaptation.

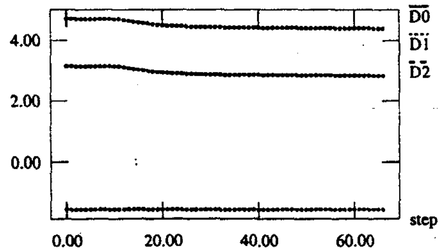


Figure 14: Change of $D_j (j = 0, 1, 2)$ by adaptation.

A framework for adaptation in rhythmic motion

Based on the above discussions, we propose a general framework of adaptation mechanism in rhythmic movements, as shown in Fig.15. Here, x denotes the state of rhythmic movement, y denotes environment, parameter λ affects rhythmic pattern and parameter μ specifies environment. In the above locomotion case, for example, the parameters are $x = [\theta_0, \theta_1, \theta_2, \theta_3]$, $y = [\rho_0, \rho_1, \rho_2, \rho_3]$, $\lambda = [\omega_0, \omega_1, \omega_2, \omega_3, D_0, D_1, D_2]$, $\mu = [\mu_0, \mu_1, \mu_2, \mu_3]$, respectively.

Environment Dynamics: The dynamics of environment varies according to parameter μ. The parameter μ must be constant for convergence of adaptation. In the locomotion case, environmental dynamics are described by eq. (8).

Rhythmic Motion Dynamics: Rhythmic motion pattern is a attractor within the space of x. It is determined by parameter λ of rhythmic motion pattern. In addition, it is also affected by the environment y. Accordingly, dynamics of x contains not only x but also y and λ. In the locomotion case, rhythmic motion dynamics are described by eqs. (2)-(7).

Adaptation Dynamics: The rhythmic motion is directly influenced from the environment. In order to adapt to the environment so as to minimize some evaluation function $E(x)$ (V_D and F_i in the locomotion case), we should adjust the parameter λ of the motion pattern. Further, the dynamics of λ must be slower enough than that of x, so that it is possible to adjust λ according to the evaluation of motion pattern. In the locomotion case, adaptation dynamics are described by eqs. (14)-(17).

In this framework, adaptation can be defined as to find suitable parameter λ of rhythmic pattern against given environment μ. This process is executed through rhythmic movement.

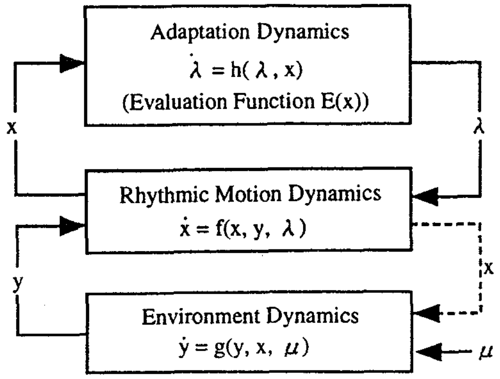


Figure 15: Our framework for adaptation in rhythmic movements.

5. CONCLUSION

This paper studied the adaptation mechanism of rhythmic movement to the environmental changes. By analyzing the perturbed locomotion, this paper proposed a mathematical framework of adaptive behavior, where the potential function of the relative phase dynamics is adjusted. This kind of adjustment minimizes the interaction between each subsystem and allows the system to memorize the motion pattern with respect to the corresponding environment. In our adaptive mechanism, it is necessary for the time scale of adaptation dynamics τ_ω and τ_D to be much larger than that of rhythmic motion dynamics $\tau_{\theta_i}(i = 0, 1, 2, 3)$. Furthermore, this paper proposed a general framework which takes into account of the environmental changes.

One of our future works will take into account the real dynamics of limbs or body. In the present paper, we implicitly assumed that phase of limb movement is equal to that of oscillator. However, they are not always equal. It will become a problem how to deal with the difference of these two phases. In addition, postural balance, because of body dynamics, will be another problem, since it is certain that the interlimb coordination is acquired through the postural balance.

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APPENDIX

Adjustment of angular velocity in a swing phase

Dynamics of oscillator in swing phase is given by

$$\dot{\theta}_i = \omega_i + f_i \qquad (i = 0, 1, 2, 3). \tag{19}$$

Integrating them during swing phase, we get

$$\theta_i(t) = \omega_i t + F_i \qquad (i = 0, 1, 2, 3), \tag{20}$$

where

$$F_i = \int_{T_{sw}} f_i dt \qquad (i = 0, 1, 2, 3), \tag{21}$$

and T_{sw} is a duration of swing phase. If $F_i > 0$ (or < 0), then θ_i is accelerated (or decelerated). In order to reduce the interaction F_i, we adjust ω_i in proportion to F_i as

$$\omega_i^{(n+1)} = \omega_i^{(n)} + \tau_\omega F_i \qquad (i = 0, 1, 2, 3) \tag{22}$$

If $F_i = 0$, then we do not change ω_i because ω_i is neither small nor large.

Since we assume the interaction do not work in a stance phase, it enables us to rewrite eq. (21) as follows:

$$F_i = \int_T f_i dt \qquad (i = 0, 1, 2, 3) \tag{23}$$