

Experiments on torque pattern learning for static balance with respect to unknown periodic external force

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Abstract: This paper considers torque pattern learning for balance control with respect to unknown periodic external forces. To cope with uncertain factor of environment, the feedback control is essential. For balancing problem, we propose a control method based on the ground reaction force feedback. When external force is periodic, the torque pattern becomes regular. Learning this torque pattern, the feedback information is less important. We show this learning process by the experiment.

Keywords: static balance, learning, motion pattern, ground reaction force, forced oscillation

1. Introduction

An application of control technology is found in the field of mechatronics, especially robotics. Because the application of robots widely spreads, a problem that conditions of robot environment cannot be described at the designing stage of robot controller arises. For such uncertainties of environment, animals like which we make robots behave certainly utilize a feedback signal. However, when the environmental conditions are clarified through repeated behaviors, animals learn appropriate motor behaviors to the current environment, and memorize them as motor patterns. Consequently, the motor behavior will be generated without less feedback information. Such a motor learning is important factor to construct robots that possesses adaptability or versatility.

Along this line, we here consider how motor control system learns motion patterns from uncertain environment by taking a static balance of upright standing as an example. The upright standing is well described as an inverted pendulum, one typical example of unstable controlled system, with small support. This smallness limits output torque at the base of the inverted pendulum: large torque makes the support rotate around its end. To express uncertainties of environment, we introduce unknown external force exerted to the inverted pendulum. Thus, the goal of this problem is to keep the balance with small base torque even under unknown external forces. This goal will be achieved by use of feedback information to the balance control. Now, assume an unknown external force possess regularity such as periodicity. In this case, humans, one of animals that can stand upright, achieve such a balancing task by learning the torque pattern for balancing from its regularities. As a result, the importance of the feedback information decreases in the controlling task. One of the purposes in our study is to describe such a learning process and to examine its feasibility by the robot motion.

For the static balance, we proposed a control method under unknown constant external force and a learning method for unknown periodic external force based on the above scenario^{1, 0, 0}. However, in these works, the period of periodic external force had to be known. In this paper, we extend the above methods to cope with unknown periodic external force with unknown period. As a result, the static balance is kept with acquired torque pattern that contains no feedback information of the ground reaction force.

This paper is organized as follows: in the second section, we review our studies for static balance control and its torque pattern learning, and clarify the problems we left. In the third section, we propose a new method to solve it. The fourth section shows robot experiments and the last fifth section gives concluding remarks.

2. Previous studies

2.1 A model for balance control

We adopt a simple mathematical model for standing posture as shown in Fig. 1(a). This consists of two links, a body segment and a foot segment. The motion is restricted in the sagittal plane, and the symmetrical foot segment contacts the ground at only two points, i.e., toe and heel, which is the both ends of foot segment. Two kinds of sensory signals are available: ankle joint, i.e., angle θ and its velocity $\dot{\theta}$, and ground reaction force at the contact point. The information of ground reaction force is only on the vertical component, which are denoted by F_T (at the toe) and F_H (at the heel). Expressing uncertainties of environment, unknown external force exerted to the center of the body segment is introduced. The horizontal and vertical components of unknown external force is denoted by F_x and F_y , respectively.

The motion equation of this model is described as

$$\begin{aligned} I\ddot{\theta} &= MLg \sin \theta + F_x L \cos \theta - F_y L \sin \theta + \tau, \\ &= F_A L \sin(\theta - \theta_f) + \tau \end{aligned} \quad (1)$$

where I is the inertial moment of the body segment around the ankle joint, M is its mass, L is the length between the ankle joint and its COG, g is the gravitational acceleration, F_A is a magnitude of resultant force exerted to the COG of the body segment that is calculated as

$$F_A = \sqrt{(Mg - F_y)^2 + F_x^2}, \quad (2)$$

and

$$\tan \theta_f = -\frac{F_x}{Mg - F_y}. \quad (3)$$

The main important purpose here is to keep the static balance of this model with respect to this unknown external force.

2.2 Balance control with ground reaction force feedback^{1, 0)}

For this purpose, we newly introduced feedback of ground reaction forces. We constructed an ankle joint torque τ so that the two ground reaction force F_H and F_T become the same at the stationary state:

$$\tau = -K_d \dot{\theta} - K_p \theta + K_f \int (F_H - F_T) dt. \quad (4)$$

where, K_d , K_p and K_f are feedback gains. If the unknown external force is constant, a posture where the moment produced by the external force is cancelled by the gravitational force, as shown in Fig. 1(b), becomes stationary. The ankle joint angle then is $\theta = \theta_f$. Furthermore, based on the linearization around the steady state, this posture becomes locally asymptotic stable if the feedback gains hold the following inequalities:

$$K_p > F_A L > 0 \quad (5)$$

$$\frac{\ell}{I} K_d > K_f > 0 \quad (6)$$

$$(K_d \ell - K_f I) K_p > K_d \ell F_A L \quad (7)$$

Here, ℓ is the distance from ankle joint to the end of the foot segment.

In summary, the control law (4) achieves the balance control with respect to unknown external force, uncertain factors of the environment. The stationary posture of this control law alters with the unknown external force, since the θ_f depends on the magnitude of F_x and F_y . The position of CoP (i.e., ZMP)⁰⁾ goes to the middle of the foot segment and then the ankle joint torque becomes zero as the moment from gravity and external force cancel out each other.

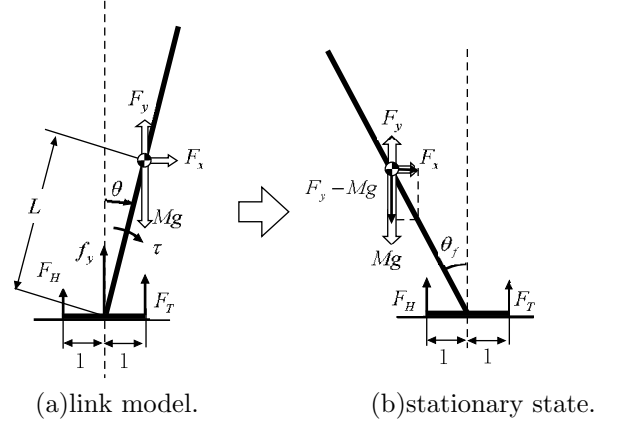


Figure 1: Model and stationary posture by control law (4).

2.3 Torque pattern learning for periodic external force^{0, 0)}

In the control law (4), the feedback information on ground reaction force is essential to cope with unknown external force. However, when the unknown external force is constant, the feedback information is unnecessary since the ankle joint torque is zero. It indicates that, if the environment is stationary, static balance is maintained without feedback information on ground reaction force. Now, we extend this idea to dynamically stationary case, i.e., unknown external force is periodic. At the first step, the period of the unknown external force T_e is assumed known.

We constructed the torque as the summation of two term: feedforward term [F.F.], and feedback term including information on ground reaction force.

$$\tau = [\text{F.F.}] + \left[-K_d \dot{\theta} - K_p \theta + K_f \int (F_H - F_T) dt \right] \quad (8)$$

Note that the second feedback term is the same as (4), owing to which the balancing is expected to be maintained regardless of unknown factors.

Next, we proposed a learning law for the first term. The criterion of the learning was to replace the second term by the learned first term. From the assumption, periodic external force is expanded as Fourier series:

$$F_x = \sum_k^n \left\{ \alpha_k^{(x)} S_k + \beta_k^{(x)} C_k \right\} \quad (9)$$

$$F_y = \sum_k^n \left\{ \alpha_k^{(y)} S_k + \beta_k^{(y)} C_k \right\} \quad (10)$$

where, $S_k = \sin k\omega_e t$, $C_k = \cos k\omega_e t$ and $\omega_e = 2\pi/T_e$. Substituting (1) by (9) and (10), we obtain

$$\begin{aligned} I\ddot{\theta} - MLgS - \sum_k^n \left\{ \alpha_k^{(x)} S_k + \beta_k^{(x)} C_k \right\} LC \\ + \sum_k^n \left\{ \alpha_k^{(y)} S_k + \beta_k^{(y)} C_k \right\} LS = \tau \end{aligned} \quad (11)$$

Here, $C = \cos \theta$ and $S = \sin \theta$. Furthermore, the right hand side of this equation is expressed as a linear form for unknown parameter vector σ :

$$Y\sigma = \tau \quad (12)$$

$$Y = \begin{bmatrix} \ddot{\theta}, S, S_0C, C_0C, S_0S, C_0S, \\ \dots, S_nC, C_nC, S_nS, C_nS \end{bmatrix} \quad (13)$$

$$\sigma = \begin{bmatrix} I, -MgL, -L\alpha_0^{(x)}, -L\beta_0^{(x)}, L\alpha_0^{(y)}, L\beta_0^{(y)}, \\ \dots, -L\alpha_n^{(x)}, -L\beta_n^{(x)}, L\alpha_n^{(y)}, L\beta_n^{(y)} \end{bmatrix}^T \quad (14)$$

Here we define a new parameter vector ϕ as

$$\phi = K_I \sigma \quad (15)$$

where

$$K_I = \frac{K_d \ell}{K_d \ell - K_f I} \quad (16)$$

and, with its estimates $\hat{\phi}$, we give a control law as follows

$$\tau = Y_r \hat{\phi} - K_d s \quad (17)$$

$$Y_r = \begin{bmatrix} \ddot{\theta}_r, S, S_0C, C_0C, S_0S, C_0S, \\ \dots, S_nC, C_nC, S_nS, C_nS \end{bmatrix} \quad (18)$$

$$\dot{\theta}_r = -\frac{K_p}{K_d} \theta \quad (19)$$

$$s = \dot{\theta} - \dot{\theta}_r - \frac{K_f}{K_d} \tau_f \quad (20)$$

In addition to this control law, we define a learning dynamics of parameter ϕ as

$$\dot{\hat{\phi}} = -\Gamma Y_r^T s \quad (21)$$

where, Γ is a positive definite diagonal matrix.

According to the above learning law (21), the second term of (17) goes to zero. This is shown by Lyapunov like lemma⁹⁾ with Lyapunov function candidate

$$V = \frac{1}{2} (K_I s^2 + \bar{\phi}^T \Gamma^{-1} \bar{\phi}) \quad (22)$$

As a result, the feedback term replaced by the feedforward term by learning, and the torque generation turns to be independent of the information on the ground reaction force.

3. Extension to external force with unknown period

The learning is essentially an estimation of external force described by the Fourier series. Then, the period, or angular frequency ω_e , of the external force must be

known, since the basis functions of Fourier series are sinusoidal functions with ‘known’ basic frequency. However, the period of unknown external force is usually unknown too. Thus, this formulation is not available in the normal way for general unknown periodic external force.

To solve this problem, we propose a method for estimating the angular frequency ω_e , more precisely, for generating $\sin k\omega_e t$ and $\cos k\omega_e t$, based on the oscillation forced by the external force. When the standing posture is maintained, the motion of the ankle joint, i.e., $\dot{\theta}$ should change with the same period as that of the external force. This implies that, by observing the ankle joint motion, the period of unknown external force may be detected. Thus, we construct a passive oscillator element inside the controller and make it interact with the external force. If the entrainment happens between them, the internal passive oscillatory element is synchronized with the external force.

The oscillation is generally represented by three parameters: amplitude, frequency and phase lag. To estimate these three parameters, at least three equations are necessary. Thus, we describe the internal oscillatory element as the differential equation, whose state variables are more than three, with the forced input from the ankle joint motion $\dot{\theta}$:

$$x^{(n)} + p_{n-1}x^{(n-1)} + \dots + p_0x = q\dot{\theta}, \quad (n \geq 3). \quad (23)$$

Here, $x^{(i)}$ is the i -th order derivative of x , and p_i and q is a constant.

The above equation works as the n -th order low-pass filter for appropriate parameters. Thanks to this low-pass property, we here assume that the lowest frequency component, i.e., basic frequency of the external force should be obtained. Describe it as

$$x = x_A \sin(\omega_e t + \alpha) \quad (24)$$

and then three of state variables of internal oscillation, i.e., $(x_0, x_1, x_2) = (x, x^{(1)}, x^{(2)})$ are theoretically described as

$$x_0 = x_A \sin \xi \quad (25)$$

$$x_1 = x_A \omega_e \cos \xi \quad (26)$$

$$x_2 = -x_A \omega_e^2 \sin \xi \quad (27)$$

Here, $\xi = \omega_e t + \alpha$. Solving the above three equations to three unknown parameters x_A , ω_e and ξ , we calculate $\sin \xi$ and $\cos \xi$ that correspond to the basic oscillation. The harmonic components are obtained by the recurrence formula:

$$\begin{bmatrix} \cos n\xi \\ \sin n\xi \end{bmatrix} = \begin{bmatrix} \cos \xi & -\sin \xi \\ \sin \xi & \cos \xi \end{bmatrix} \begin{bmatrix} \cos(n-1)\xi \\ \sin(n-1)\xi \end{bmatrix} \quad (28)$$

By the way, because the Fourier expansion is applied to only the unknown external force, Y in (12) includes not only S_k and C_k but also θ and $\dot{\theta}$. However, the behavior of balancing motion adjusting with periodic external force should be also periodic. In addition, its

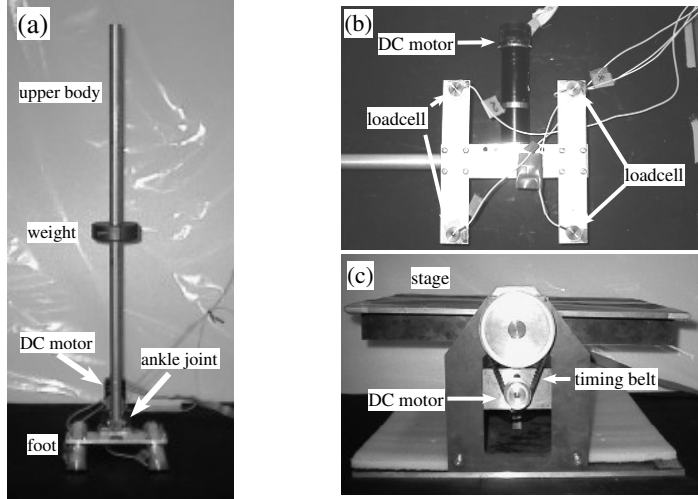


Figure 2: Apparatus. (a) Balancing robot. (b) sole with loadcells. (c) slope stand.

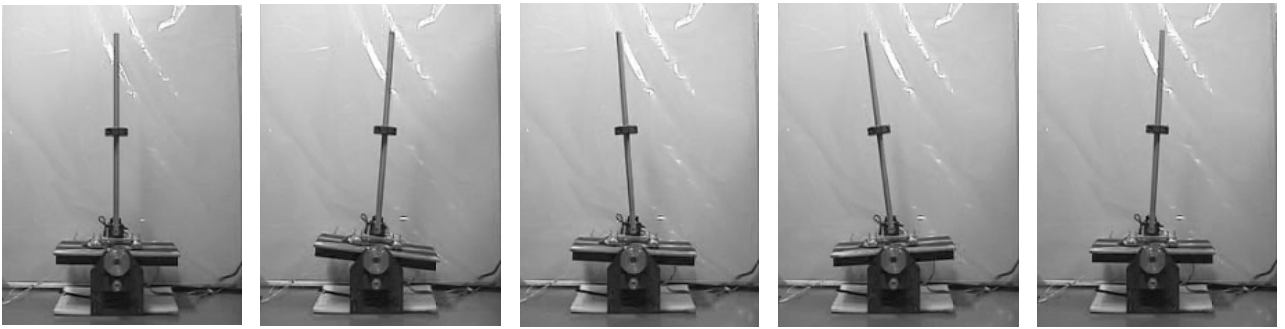


Figure 3: Photos of robot experiment on swaying slope stand.

period should be the same as that of the periodic external force. This indicates that θ as well as $\dot{\theta}$ are also expanded to Fourier series. Consequently, the whole left hand side of (11) can be described as the Fourier series consisting of the fundamental basis function $\sin \xi$ and $\cos \xi$. Thus, in (11), we define Y as

$$Y = [\sin \xi, \cos \xi, \dots, \sin n\xi, \cos n\xi] \quad (29)$$

and σ as a column vector of Fourier coefficients for corresponding frequency, instead of (13) and (14). Because this formulation gives no effect to the following analysis, we utilize Y_r derived from (29) for the control and learning.

4. Experiments

4.1 Setups and conditions

A simple two-link robot, as illustrated in Fig. 2(a), is used to verify the theoretical framework in the previous section. The body segment is 0.5[m] and the foot segment is 0.1[m] length. A 20[W] DC servo-motor with 53:1 reduction gear is installed 0.046[m] height at the middle of the foot segment. This motor is directly connected to the body segment. As shown in Fig. 2(b),

four force sensors are attached at the corner of the sole. They detect the vertical components of the ground reaction F_T and F_H by summing up two of them, respectively. The ankle joint angle is detected by the rotary encoder installed at the motor of the ankle joint.

The passive internal oscillatory element is prepared based on the 3rd-order Butterworth filter. Because the period of the external force is unknown, the break frequency cannot be set in advance. However, we here restrict the problem to balance control of the robot: The external force around 1[Hz]-order basic frequency is very rare. Even if such a high-frequency force were exerted, the balance would be maintained in a feedback manner due to its rapid alterations and thus, in our opinion, the learning from its periodicity would not be observed in the real balancing task. On the other hand, for a slow external force less than 0.01[Hz], an intermittent feedback control will be sufficient and so it is not necessary to learn motion pattern. Consequently, we consider that the learning is effective for external force around 0.1[Hz] basic frequency. This is a reason why we set the break frequency of the filter at 0.2[Hz].

To solve the equations (26)–(27), we calculate ξ at

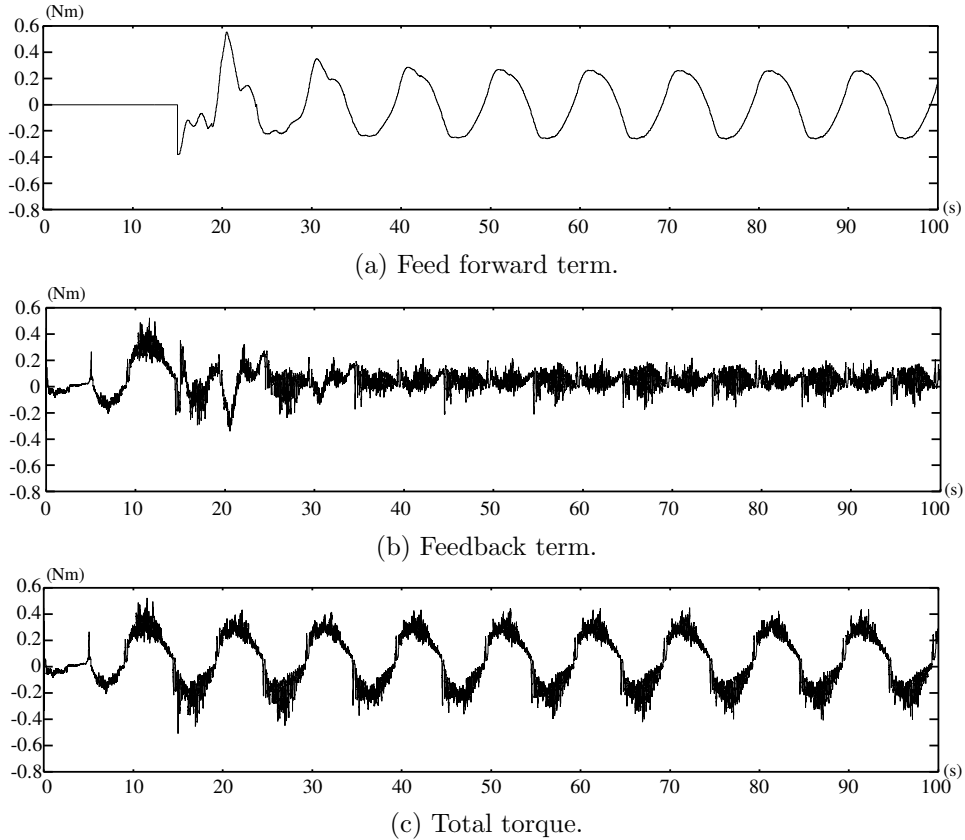


Figure 4: Balance control experiment under 0.1Hz periodic external force.

first from the relation

$$\xi = \text{atan2}(x_1, \text{sgn}(x_0)\sqrt{-x_0x_2}) \text{ (if } x_0x_2 \leq 0) \quad (30)$$

and then compute $\sin \xi$ and $\cos \xi$. Here, $\text{sgn}(x)$ is a function that returns the sign of the argument x . In the experiments, however, the case where $x_0x_2 > 0$ appears. In this case, we keep ξ at the current value.

To provide stationary environmental conditions, we construct the slope stand driving by DC servo-motor, as shown in Fig. 2(c). Instead of generating the periodic external force, we periodically vary the gradient of the slope stand. The amplitude of the gradient variation is 0.12[rad], and two frequencies are set, 0.1[Hz] and 0.2[Hz]. The parameters are set as follows: $K_d = 12$, $K_p = 70$, $K_f = 2$, $n = 8$, $\Gamma = [5, \dots, 5]$. The 3rd-order Butterworth filter is adopted as a internal oscillatory element and its state variables are computed by numerical integration followed by 4th order Runge-Kutta method. In the experiments, a 0.415[kg] weight is attached at the body segment 0.25[m] height from the ankle joint to adjust the center of mass height. The control and learning interval is set to 2[msec] and the duration of experiments is 100[sec]. So as to stabilize the robot motion, the learning of the feedforward term turns effective 15[sec] after the start of the experiment

4.2 Results

The robot adjusted the ankle joint to prevent itself from tumbling on the moving slope stand, as depicted in Fig. 3. The experimental results are illustrated in Fig. 4 and Fig. 5, where frequency of the moving slope stand is respectively 0.1[Hz] and 0.2[Hz]. In each figures, (a) shows the feedforward component of ankle joint torque, (b) does the feedback one and (c) does the total one which is provided as the motor command. Although small high-frequency vibration are left as unlearned component, the adequate torque profile tend to be stored in memory independent of frequencies of external forces.

5. Conclusion

In this paper, we extend our control and learning method for static balance maintenance to cope with unknown periodic external force with unknown period. As a result, the static balance is kept with acquired torque pattern that almost never contains feedback information of the ground reaction force. The main issue here is how to construct the basis functions of Fourier series expansion for describing environment by observing the effect of external force. For this purpose, we introduce a passive oscillatory element inside the controller and make it forcedly oscillate with ankle joint motion that varies with external force. This passive oscillatory el-

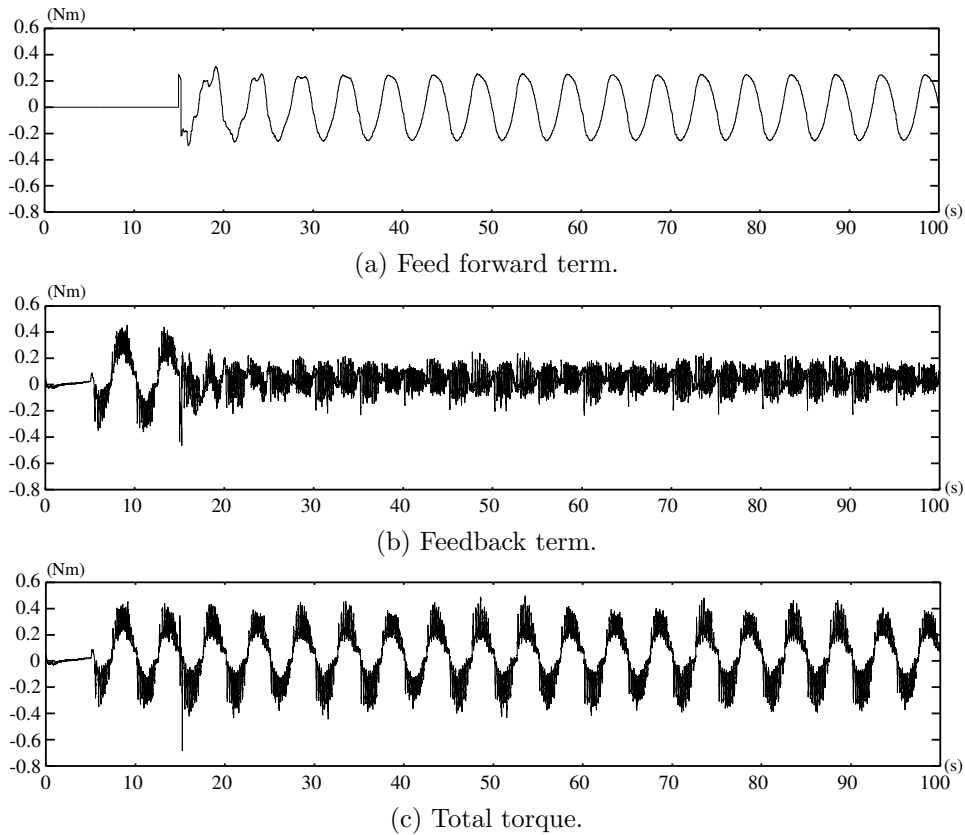


Figure 5: Balance control experiment under 0.2Hz periodic external force.

ement works as a low-pass filter and consequently the basic oscillation of periodic external force is detected. The basis functions are generated based on this scheme, and the Fourier coefficients of this basis functions are estimated followed by the previous works. This scheme is illustrated as in Fig. 6. In robot experiments, we put a simple robot on the slope stand whose slope angle varies periodically. We confirmed that the torque pattern is learned to decrease the feedback term of the ground reaction force in the torque component, regardless of the frequency of the variation of the slope stand.

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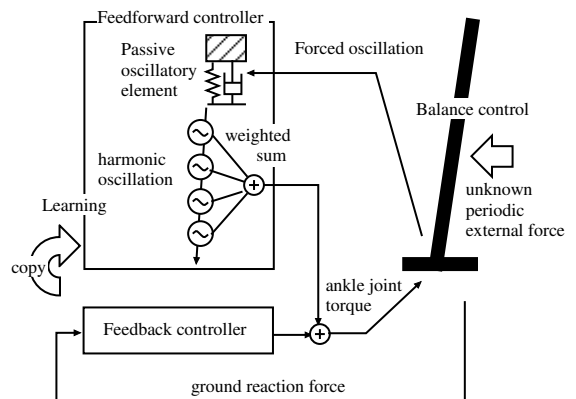


Figure 6: Control scheme.

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