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A static balance control under periodic external force

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Abstract: In this paper, we present a control method of upright posture under periodic external forces with known period. First, the upright posture is maintained based on the feedback of ground reaction forces. However, being exposed to such a stationary condition for a while, external forces are estimated using the framework of adaptive control. Consequently, the balance maintenance is achieved without feedback of ground reaction forces.

Keywords: static balance, posture control, ground reaction force, periodic external force, adaptive control

1. Introduction

To keep upright posture is a fundamental control problem for biped system. If all the environmental conditions are known, the posture such that the CoG (center of gravity) of the whole body comes above the foot part can be set as the desired posture in the designing stage of motion pattern. Thus, to avoid falling down, the positional feedback control for achieving such posture is adopted as one possible method. However, when environment contains unknown factors or dynamically changes, this control strategy will sometimes fail to keep upright posture: the posture that is designed in advance is not sufficient for the current environmental conditions. For example, the desk lamp that stably stands on the level desktop often falls over when the desk tips up. In such situations, the position of the CoG should be adjusted with the conditions of the environment. Then, the information on the current environment has to be detected, in some fashion, to know differences from the designing stage, to evaluate the stability of the body, and to output the control torque for compensating the deviation of the balance originated from unknown factors.

Among much sensory information, ground reaction forces provide the useful information for balancing, because the center of pressure (CoP) of ground reaction forces coincide to zero moment point [1)] that is widely utilized for dynamic control of walking robot [2)]. From this point of view, we have proposed a control method by actuating ankle joint based on the ground reaction forces [3, 4)]. Here, the environment is described as an unknown constant external force, and the control method makes the body part face to the direction of the resultant force of the gravity and the external force at the stationary posture. This implies not only that the stationary state changes with the external force, i.e., environmental conditions, but also that the output of the ankle joint become zero to keep this posture owing to the balance of gravity and external force.

In this paper, we extend this control method so that it can be applied to the environment with periodic external forces. Because the locomotion shows the periodic bodily movements, the dynamic property of interaction force between links should be also periodic. Therefore, if the periodic external forces are treated well, it is possible to apply it to the dynamic locomotion control

2. Upright posture control under constant external force

In this section, we review the control method of upright posture which was proposed in the previous papers[3, 4)], since it become the basis for the control law in the present paper.

Throughout this paper, we consider a 2-link model in the sagittal plane which contacts to the ground at the two points, i.e., at the both tips of the foot part, as shown in Fig. 1(a). Here, we assume that the foot part possesses a symmetrical shape in the anterior-posterior direction as well as the ankle joint is located just above the ground surface. When the static balance is kept, the foot part does not move and so does not have dynamics. From the balance of the moment around the contact points, we can obtain the relationship between the ankle joint torque and the ground reaction forces:

$$F_H = \frac{1}{2\ell}\tau + \frac{1}{2}mg + \frac{1}{2}f_y, \quad (1)$$

$$F_T = -\frac{1}{2\ell}\tau + \frac{1}{2}mg + \frac{1}{2}f_y. \quad (2)$$

Here, F_H and F_T are the vertical component of the ground reaction forces at the contact points respectively, m is mass of the foot part, f_y is force acting from the body part that is given as

$$f_y = -ML\ddot{\theta}\sin\theta - ML\dot{\theta}^2\cos\theta + Mg - F_y. \quad (3)$$

Subtracting (2) from (1), we get the relation between the ankle joint torque and difference of the two ground reaction force, $F_H - F_T$

$$F_H - F_T = \frac{1}{\ell}\tau. \quad (4)$$

On the other hand, the motion equation of the body part is described as that of the inverted pendulum:

$$\begin{aligned} I\ddot{\theta} &= MgL\sin\theta + F_xL\cos\theta - F_yL\sin\theta + \tau, \\ &= AL\sin(\theta - \theta_f) + \tau \end{aligned} \quad (5)$$

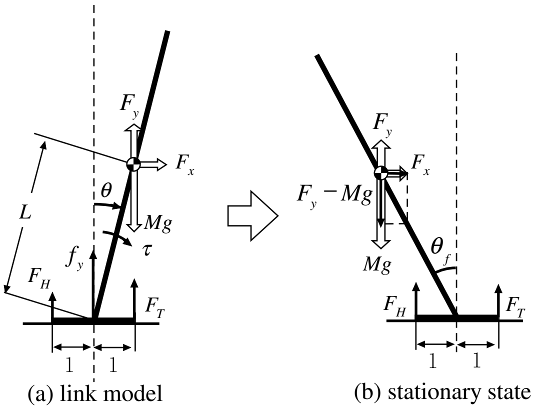


Figure 1: Model and stationary posture by proposed control law.

where M is mass of the body part, I is moment of inertia of the body part around ankle joint, L is distance from the ankle to CoG of the body part, θ is a sway angle of the body part, τ is ankle joint torque, g is gravitational acceleration, F_x and F_y are external force which is assumed constant here, and A and θ_f are constants that satisfy the following equations:

$$A = \sqrt{(Mg - F_y)^2 + F_x^2} \tag{6}$$

$$\sin\theta_f = -\frac{F_x}{A}, \ \ \cos\theta_f = \frac{Mg - F_y}{A}. \tag{7}$$

Under these problem setting, we define the ankle joint torque τ as the following equation to make F_T and F_T take the same value without falling down:

$$\tau = -K_d\dot{\theta} + K_p(\theta_d - \theta) + K_f \int (F_H - F_T)dt. \tag{8}$$

This control torque produces the stationary posture in which the gravity and external force are balanced, as illustrated in Fig. 1(b). The stability of this posture is locally ensured by the following equations on the feedback gains, K_d, K_p and K_f:

$$K_p > AL > 0 \tag{9}$$

$$\frac{\ell}{I} K_d > K_f > 0 \tag{10}$$

$$(K_d\ell - K_f I)K_p > K_d \ell AL \tag{11}$$

where ℓ is the distance from ankle joint to the tip of the foot part. In equations, $\theta = \theta_f$ becomes the local stable equilibrium point of the dynamics defined by (4) and (5), and $F_H = F_T$ holds at this stable equilibrium point.

3. Upright posture control under periodic external force

3.1 strategy

The characteristic of the control method in the previous section is represented in the feedback of the ground reaction forces. This method is equivalent to the feedback control of CoP. It should be noted that ankle joint torque becomes zero at the stationary state, since the moment of gravity and external force is balanced around it. This implies that the feedback of ground reaction force is not necessary any more, once the stationary state is achieved.

From this point of view, we aim at a posture control method under the periodic external force such that the feedback of ground reaction force is unnecessary after adaptive learning. Thus, we construct the ankle joint torque as the summation of the two components: the one compensating the periodic external force in a feedforward manner and the one at right hand side of (8) that includes the feedback of ground reaction forces, like

$$\tau = [F.F] + \left[-K_d\dot{\theta} - K_p\theta + K_f \int (F_H - F_T)dt\right] \tag{12}$$

For the learning of the feedforward components, we adopt the framework of the adaptive control proposed by J-J. E. Slotine and W. P. Li[5)], which produces the first term of the right hand side in (12) so that the second term converges to zero. Here, it is important to define the first term so as not to contain any feedback information of ground reaction forces.

3.2 Linearization for unknown parameters

Throughout this paper, we assume the period of the periodic external force T_e is known. Under this assumption, the external forces are expanded to the Fourie series whose basic frequency is the same as that of the external force:

$$F_x = \sum_k^n \left\{\alpha_k^{(x)} S_k + \beta_k^{(x)} C_k\right\} \tag{13}$$

$$F_y = \sum_k^n \left\{\alpha_k^{(y)} S_k + \beta_k^{(y)} C_k\right\} \tag{14}$$

Here, $S_k = \sin k\omega_e t$, $C_k = \cos n\omega_e t$ and $\omega_e = 2\pi/T_e$. Substituting (13) and (14) into (5), we obtain

$$I\ddot{\theta} - MgLS - \sum_k^n \left\{\alpha_k^{(x)} S_k + \beta_k^{(x)} C_k\right\} LC + \sum_k^n \left\{\alpha_k^{(y)} S_k + \beta_k^{(y)} C_k\right\} LS = \tau \tag{15}$$

To simplify the notations, we put $C = \cos\theta$ and $S = \sin\theta$. We can linearize the left hand side of the above equation for the unknown parameters:

$$Y\sigma = \tau \tag{16}$$

$$Y = \left[\ddot{\theta}, S, S_0C, C_0C, S_0S, C_0S, \cdots, S_nC, C_nC, S_nS, C_nS\right] \tag{17}$$

$$\sigma = \left[I, -MgL, -L\alpha_0^{(x)}, -L\beta_0^{(x)}, L\alpha_0^{(y)}, L\beta_0^{(y)}, \cdots, -L\alpha_n^{(x)}, -L\beta_n^{(x)}, L\alpha_n^{(y)}, L\beta_n^{(y)}\right]^T \tag{18}$$

Here, σ is a vector composed of the unknown parameters, and Y corresponds to the regressor.

3.3 Control and adaptation law

To evaluate whether the falling-down occurs or not, we have to consider (4) as well as (16). For these equations, we define the ankle joint torqe using $\hat{\sigma}$, the estimates of the unknown parameter σ.

$$\tau = \frac{K_d \ell}{K_d \ell - K_f I} Y_r \hat{\sigma} - K_d s \tag{19}$$

Here, Y_r is a known column vector defined by

$$\begin{aligned} Y_r \quad = \quad & \left[\ddot{\theta}_r, S, S_0 C, C_0 C, S_0 S, C_0 S, \right. \\ & \left. \cdots, S_n C, C_n C, S_n S, C_n S\right], \end{aligned} \tag{20}$$

as $\dot{\theta}_r$, a reference velocity is constructed by

$$\dot{\theta}_r = -\frac{K_p}{K_d}\theta \tag{21}$$

Meanwhile, s is a variable defined by the following equation:

$$s = \dot{\theta} - \dot{\theta}_r - \frac{K_f}{K_d}\tau_f, \tag{22}$$

where τ_f is a new state variable composed from the feedback of the ground reaction force,

$$\tau_f = \int (F_H - F_T) dt \tag{23}$$

Note that $-K_d s$, the second term of (19) is equal to (8).

In addition to the above control law, the adaptation law is defined as

$$\dot{\hat{\sigma}} = -\Gamma Y_r^T s \tag{24}$$

For the control law (19) and adaptation law (24), we here show that s converges to zero. As a candidate of the Lyapnov function, we consider the following function:

$$V = \frac{1}{2} I s^2 + \frac{1}{2}\bar{\sigma}^T \Gamma^{-1} \bar{\sigma} (\geq 0) \tag{25}$$

where $\bar{\sigma} = \hat{\sigma} - \sigma$. Differentiating (25) by time, we obtain

$$\dot{V} = I s \dot{s} + \dot{\hat{\sigma}}^T \Gamma^{-1} \bar{\sigma} \tag{26}$$

From the definition of Y_r, the next equation is satisfied:

$$\begin{aligned} I\ddot{\theta}_r - MLgS - \sum_k^n \left\{ \alpha_k^{(x)} S_k + \beta_k^{(x)} C_k \right\} LC \\ + \sum_k^n \left\{ \alpha_k^{(y)} S_k + \beta_k^{(y)} C_k \right\} LS = Y_r \sigma \end{aligned} \tag{27}$$

Subtracting (27) from (15), we can get

$$I(\ddot{\theta} - \ddot{\theta}_r) = \tau - Y_r \sigma. \tag{28}$$

On the other hand, (23) turns to

$$\dot{\tau}_f = \frac{1}{\ell}\tau \tag{29}$$

by substituting (4) after differentiation. Next, subtracting IK_f/K_d times of (29) from (28)

$$I(\ddot{\theta} - \ddot{\theta}_r - \frac{K_f}{K_d}\dot{\tau}_f) = (1 - \frac{IK_f}{K_d \ell})\tau - Y_r \sigma \tag{30}$$

Then, substituting (22), (21) and the control law (19), we finally obtain

$$I\dot{s} = Y_r \bar{\sigma} - K_V s \tag{31}$$

Here,

$$K_V = (K_d \ell - K_f I)/\ell. \tag{32}$$

From (31), (26) becomes

$$\begin{aligned} \dot{V} = & \quad sY_r\bar{\sigma} - K_V s^2 + \bar{\sigma}^T \Gamma^{-1} \dot{\hat{\sigma}} \\ = & \quad \bar{\sigma}^T (Y_r^T s + \Gamma^{-1} \dot{\hat{\sigma}}) - K_V s^2. \end{aligned} \tag{33}$$

However, by using the adaptation law, (24), it finally turns to

$$\dot{V} = -K_V s^2 \leq 0. \tag{34}$$

Here, K_V becomes positive if the feedback controller based on ground reaction forces is constructed to satisfy the condition (9) - (11).

To prove that $\dot{V}$ converges to 0, we next show the uniformly continuity of $\dot{V}$. All we have to do is to show the boundedness of $\ddot{V}$

$$\ddot{V} = -2K_V s\dot{s} \tag{35}$$

From the fact that $V \geq 0$ and $\dot{V} \leq 0$, V is bounded. It indicates s and $\bar{\sigma}$ are bounded. The boundedness of s leads to the boundedness of θ, $\dot{\theta}$ and τ_f, while the boundedness of $\bar{\sigma}$ does to the boundedness of $\hat{\sigma}$. Based on these boundedness, we can show the boundedness of $\dot{\theta}_r$ from (21) and the boundedness of Y_r from (20). In addition, $\dot{s}$ is bounded since $I \neq 0$ in (28). Because of the boundedness of s and $\dot{s}$, $\ddot{V}$ becomes bounded.

Now, using Lyapnov like lemma[5)], $\dot{V}$ converges to 0, which is equivalent that $s \to 0$ in the stationary state.

3.4 Remarks

First of all, the feedback gain of the second term of (19) should satisfy (9)–(11), which implies that the static balance is kept under the constant external force, i.e., the special case of periodic external force with infinite period. Next, in order that the adaptation law (24) works continuously, the balance also continues to be kept without falling down. However, because the stability is $\hat{\sigma}$ only locally, a large external force may make it fall down. The acceptable amplitude of the periodic external force will be estimated by the (8).

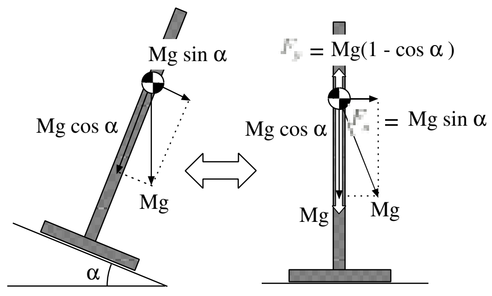
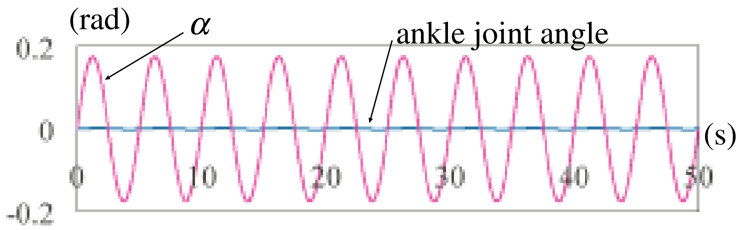
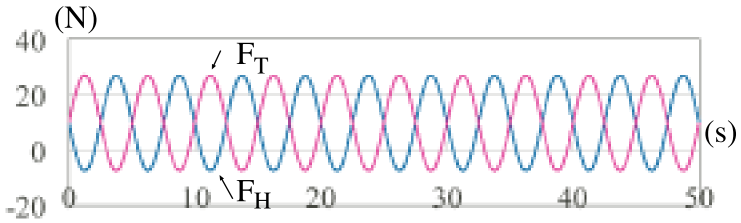


Figure 2: Definition of external force.



(a) Slope and ankle joint angles.



(b) Ground reaction forces.

Figure 3: Results with PD control.

When the stationary state is achieved with this control law, the ankle joint torque (8) is reconstructed by the weighted sum of the basis function consisting of each element of Y_r, whose weights, i.e, $\hat{\sigma}$ are learned according to the adaptation law (24). Because each element of Y_r does not contain the feedback of the ground reaction forces, the postural balance can be maintained without them. If we redefine $\hat{\sigma}$ including an unknown parameter $K_d\ell/(K_d\ell - K_f I)$ in (19), the control law can be constructed without unknown parameters.

Suppose that the maintenance of balance is achieved only with the second term of (19). This control law is described with the condition $\hat{\sigma}(0) = 0$ and $\Gamma = 0$, and is equivalent to the control law (8). This enable us to infer that the balance is also kept even if the adaptation law started by setting $\Gamma \neq 0$, because the feedback control of ground reaction force is still effective on the background of the adaptation law. However, while we have shown that $s \to 0$ according to the adaptation law, we have not proved yet that this adaptation law never make it fall down, mathematically. Regarding to this issue, we will show the validness by the computer simulations at the next section.

4. Simulation

Using the 2-link model in Fig. 1, we executed computer simulations. The parameters of the link model are set, $M = 2$(kg), $L = 0.5$(m), $\ell = 0.05$(m), $I = 5ML^2/4$(kgm^2). In order to compare the control laws, we examined tree cases: (A) only conventional PD control, (B) (8) that contains the feedback of ground reaction forces, and (C) (19) – (24) that contain adaptation law as well as the feedback of ground reaction forces. We define the periodic external force with the period 5 (s) as

$$F_x = Mg\sin\alpha \tag{36}$$

$$F_y = Mg(1 - \cos\alpha) \tag{37}$$

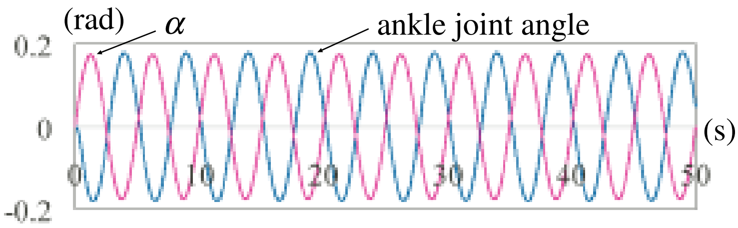
$$\alpha = \frac{\pi}{18}\sin 2\pi f_e t \qquad (f_e = 0.2(Hz)). \tag{38}$$

This external force is equivalent to the one that is exerted on the slope with the gradient α, as illustrated in Fig.2. Namely, this simulation corresponds to the situation where the gradient of the floor is oscillate from $-\pi/18$ to $\pi/18$ with the period 5 (s). The gains of the control law and other parameters are set as follows: (A) $K_d = 500$, $K_p = 1000$, (B) $K_d = 500$, $K_p = 1000$, $K_f = 25$, and (C) $K_d = 500$, $K_p = 1000$, $K_f = 25$, $\Gamma = diag[0.1, \cdots, 0.1]$, $n = 10$. The results are depicted respectively in Fig. 3, Fig. 4 and Fig. 5.

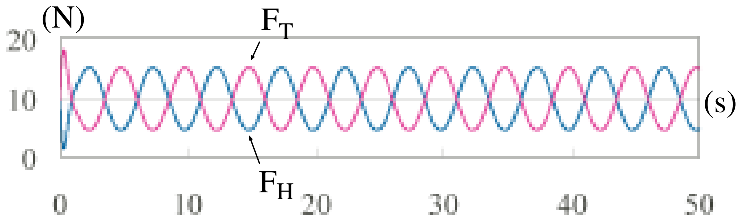
In the case of (A), the ankle joint angle is almost kept to 0 and the body usually faces to the orthogonal direction against the floor, because of the high feedback gains, as shown in Fig. 3(a). But, the ground reaction forces depicted in Fig. 3(b) sometimes take negative values, implying that the PD control practically makes the link system fall down around the tip of the foot part.

On the other hand, when the feedback of ground reaction forces added in the case (B), the ankle joint is adjusted according to the periodic external forces. As shown in Fig. 4(a), the ankle joint angle becomes almost the same as α because we define the external force so that the angle made by the ground surface and resultant force of gravity and external force becomes α. Such adjustment of ankle joint angle prevents the ground reaction forces from taking the negative values, as shown in Fig. 4(b). This indicates that the position of CoG of the body part is appropriately shifted based on the ground reaction forces, which avoids falling down even in the unknown environment.

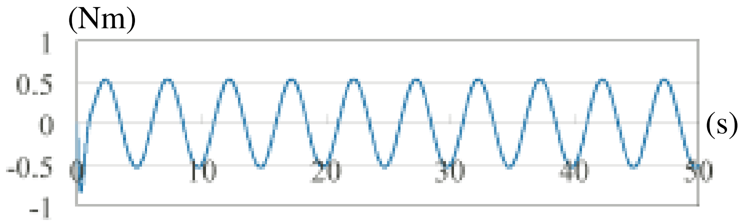
Furthermore, the adaptive learning is added in the last simulation. As depicted respectively in Fig. 5(a), (b) and (c), the time course of ankle joint angles, ground reaction forces and total ankle joint torques dose not change so much, indicating



(a) Slope and ankle joint angles.



(b) Ground reaction forces.



(c) Ankle joint torque.

Figure 4: Results with ground reaction force feedback.

that, even if the adaptation law works, the upright posture can be maintained in almost the same manner as the case without it. However, the components that construct the ankle joint torque gradually change, as shown in Fig. 5(c): The second term that contains the feedback of the ground reaction forces are decreaseing, while the first term without them are increase to finally dominate the ankle joint torque.

These three simulations provide two assertions: First, the information of the ground reaction forces are essential to keep the upright posture under the environment that contains unknown factors. Second, even though environment contains unknown factors, if it is stationary, the upright posture comes to be maintained without the feedback of ground reaction force. This is achieved by adaptive learning, which clarifies unknown factors on the stationary environment.

5. Conclusion

In the present paper, we consider an upright posture control of biped model under periodic external force with known period. In order to maintain upright posture against unknown external force, it is effective to construct a control law based on ground reaction forces. However, if the external force is periodic and its period is known, the upright posture comes to be maintained without ground reaction forces, since the unknown factors on environment is clarified during the successful posture control. The control scheme is summarized to the block diagram as depicted in Fig. 6. Using ground reaction forces, the feedforward controller in a sense that it does not need the ground reaction force is learned so as to decrease the output of their feedback controller. Such a feedforward controller may be extended to a pattern generator for the locomotion.

As an example of the periodic external force, we consider the interaction force exerted from the other links during the stationary locomotion. In the general strategy of the locomotion control, the motion pattern of each link is often given at the stage of the motion planning. It implies that the period of the locomotion have been already decided at this stage. Thus, this assumption will be valid, if the control method is applied for the balance control during locomotion.

As the future works, we should consider the extension for the periodic external forces with unknown period, the application of the motion planning of the locomotion and the enhancement of the convergence of the feedback control with the ground reaction forces.

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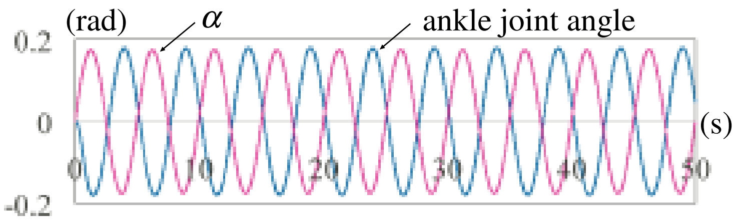
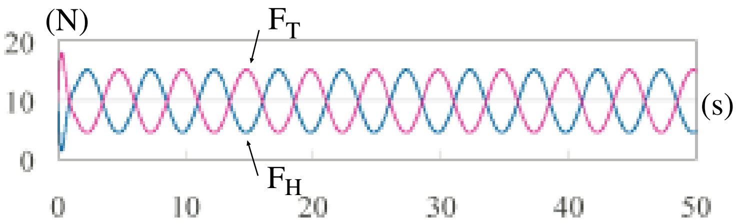
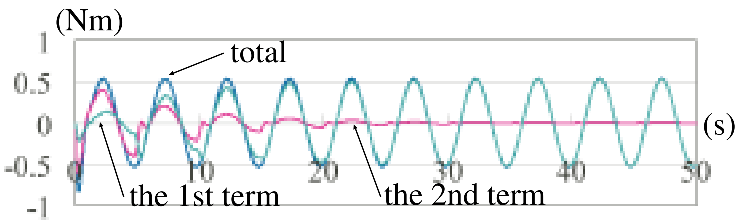


(a) Slope and ankle joint angles.



(b) Ground reaction forces.



(c) Ankle joint torque.

Figure 5: Results with ground reaction force feedback and adaptive learning.

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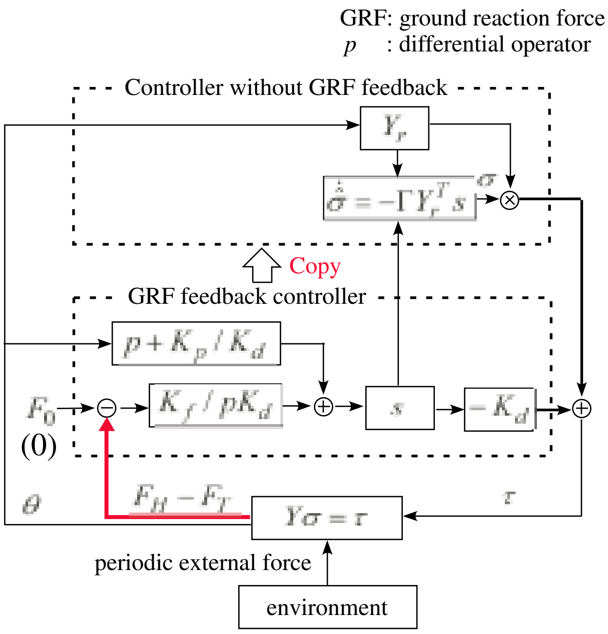

Figure 6: Block diagram of control scheme.