A consideration on position of center of ground reaction force in upright posture

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Abstract: The center of the ground reaction forces is useful to evaluate the balance of the biped system. Here, we consider where to control it when the foot has an asymmetrical structure like humaner being. Two control laws are examined from the view point of the ankle joint output and postural stability. In order to discuss which of two should be used for biped robot control, we measure the position of the center of the ground reaction forces for human upright posture.

Keywords: Biped upright posture, center of ground reaction force, human measurement, ankle joint

1. Introduction

Keeping upright posture is a fundamental and important function for the biped system. If the balance is kept statically on the level floor, the vertical projecting point of the COG (center of gravity) of the whole body must be located within a convex hull containing footprints [1, 2)]. Thus, a simple strategy to stabilize a biped robot is that a desired posture satisfying this condition is firstly designed, and next the positional feedback control is applied to achieve it.

When the ground reaction forces are detectable, they provide important information for postural balance. If statically balanced, the center of ground reaction forces coincides the vertical projecting point of the COG of the whole body [2)]. Therefore, if the center of ground reaction forces can be controlled directly, the upright posture is more robust against perturbations or modeling error than the methods depending only on the positional feedback.

In the previous paper, we proposed a control method for the ground reaction forces to stabilize the upright posture against a constant external disturbance [3, 4)]. This method was designed so that the center of ground reaction forces converges to the point just below the ankle joint in no disturbance condition. However, more detailed analysis clarifies that where the center of ground reaction forces should be controlled is more crucial problem for the stability, especially when the foot shape has no front-back symmetry or ankle joints are positioned high from the ground. In this paper, we discuss it with considering the efficiency of posture maintenance.

2. Model and control

2.1 Model of biped system

A biped standing model considered in this paper is shown in Fig. 1. Here, we focus on the function of the ankle joints, since they are located at the lowest position of the body and so give a great effect to the postural balance: the slight changes of them causes a large deviation of the COG of the whole body. Due to the right-left symmetry of the biped system, we only consider a single foot. During the upright standing, the body and legs are assumed to keep the same posture and so we model them by a single link . We deal with only the motion in the sagittal plane. To simplify the treatment of the ground reaction forces, we assume two point contacts to the ground at the both ends of the foot part. The vertical component of them is detectable, which is denoted by F_T (at the toe) and F_H (at the heel). The shape of foot is assumed asymmetry, so ℓ_T, ℓ_H and ℓ_G denotes the horizontal distance from ankle joint to toe, heel, and COG of foot respectively, and ℓ_A denotes the height of the ankle joint from the ground. The length of foot is denoted by 2ℓ, i.e. $2\ell = \ell_H + \ell_T$. The friction between foot and ground is assumed large enough to prevent the foot from slipping.

To show the effectiveness of the center of the ground reaction force control, we introduce a constant external disturbance in the horizontal direction F_x and vertical one F_y. When the static balance is kept, the foot part does not cause any motions. Thus, only the body part has dynamics, which is given by

$$I\ddot{\theta} = MLg\sin\theta + F_xL\cos\theta - F_yL\sin\theta + \tau, \qquad (1)$$

where M is the mass of the body part, I is the inertial moment of the body part around the ankle joint, L is the length between ankle joint and the COG of the body part, θ is the ankle joint angle from the vertical direction, τ is the ankle joint torque, and g is gravitational acceleration. On the other hand, from the balance of moment around the heel and toe, F_T and F_H are described,as

$$F_T = -\frac{1}{2\ell}\tau + m_Tg + \frac{\ell_H}{2\ell}f_y - \frac{\ell_A}{2\ell}f_x \qquad (2)$$

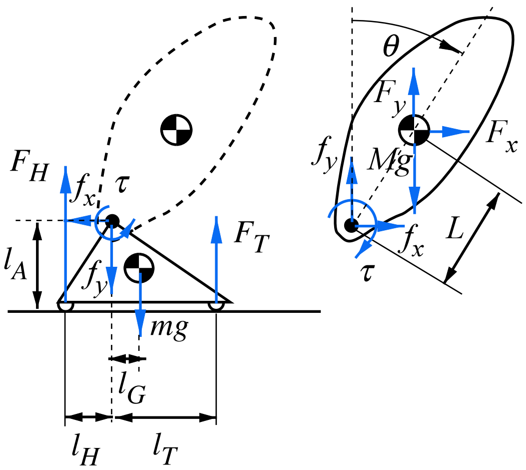


Figure 1: Biped standing model.

$$F_H = \frac{1}{2\ell}\tau + m_H g + \frac{\ell_T}{2\ell} f_y + \frac{\ell_A}{2\ell} f_x \tag{3}$$

Here, f_x and f_y are internal force between foot and body, given as

$$f_x = ML\ddot{\theta}\cos\theta - ML\dot{\theta}^2\sin\theta - F_x, \tag{4}$$

$$f_y = -ML\ddot{\theta}\sin\theta - ML\dot{\theta}^2\cos\theta + Mg - F_y. \tag{5}$$

m_T and m_H is a mass of the foot weighted respectively to the toe and heel, which is representing by

$$m_T = \frac{\ell_H + \ell_G}{2\ell} m, m_H = \frac{\ell_T - \ell_G}{2\ell} m, \tag{6}$$

where m is the total mass of the foot.

For simplicity of calculation, we transform the motion equation (1) as follows:

$$\begin{aligned} I\ddot{\theta} &= (Mg - F_y)L\sin\theta + F_x L\cos\theta + \tau \\ &= AL\sin(\theta - \theta_f) + \tau \end{aligned} \tag{7}$$

where

$$A = \sqrt{(Mg - F_y)^2 + F_x^2} \tag{8}$$

and θ_f is a constant which satisfies these equations,

$$\sin\theta_f = -\frac{F_x}{A}, \quad \cos\theta_f = \frac{Mg - F_y}{A}. \tag{9}$$

Note that A as well as θ_f depend on the constant disturbance F_x and F_y.

2.2 Control laws

Control minimizing ankle joint output When the body faces to the resultant force of the gravitational force and external force, only the small torque is necessary to keep this posture (theoretically zero). To achieve it as a stationary posture, we have proposed a control law in the previous paper [4]. This control law was given as follows:

$$\tau = -K_d\dot{\theta} + K_p(\theta_d - \theta) + K_f\tau_f \tag{10}$$

$$\tau_f = \int (F_H - F_T - F_0)dt, \tag{11}$$

$$F_0 = (m_H - m_T)g + \frac{\ell_T - \ell_H}{2\ell} f_y + \frac{\ell_A}{\ell} f_x \tag{12}$$

Throughout this paper, we call this control law, 'Control 1'. Regarding θ, $\dot{\theta}$ and τ_f as state variables, the state dynamics are described as

$$I\ddot{\theta} = AL\sin(\theta - \theta_f) - K_d\dot{\theta} + K_p(\theta_d - \theta) + K_f\tau_f \tag{13}$$

$$\dot{\tau}_f = \frac{1}{\ell}(-K_d\dot{\theta} + K_p(\theta_d - \theta) + K_f\tau_f). \tag{14}$$

To derive (14), both (2) and (3) were substituted into the time derivative of (11).

Simple calculation shows that the stationary state of this dynamics is

$$(\bar{\theta}, \bar{\tau}_f) = (\theta_f, \frac{K_p}{K_f}(\theta_f - \theta_d)). \tag{15}$$

and, it becomes local stable point if the feedback gains satisfy the following conditions [4]:

$$K_p > AL > 0, \tag{16}$$

$$\frac{\ell}{I} K_d > K_f > 0, \tag{17}$$

$$(K_d\ell - K_f I)K_p > K_d\ell AL. \tag{18}$$

Note that the body is inclined θ_f at the stationary state, which coincides the direction of the resultant force of the gravity force and constant disturbance. Apparently, the torque at the stationary state becomes zero, which is easily shown by substituting (15) to (10) with $\dot{\theta} = 0$. If the mass of the foot part is small enough to be approximated as $m \sim 0$, the center of gravitational forces is positioned at the ground crossing point by the line extended to the direction of resultant force from the ankle joint, as shown in Fig. 2,

Control maximizing stability From the aspect of stability, the center of ground reaction forces should be at the center of the foot (to the end, we denote it by point C). Unfortunately, Control 1 does not realize it. In order to enhance the stability, we here propose a modified control law such that the center of ground reaction forces converges to the point C, which we call here 'Control 2'. It will be achieved simply by put F_0 to 0 in eq (12). Then, the dynamics (14) changes to

$$\begin{aligned} \dot{\tau}_f &= \frac{1}{\ell}(-K_d\dot{\theta} + K_p(\theta_d - \theta) + K_f\tau_f) \\ &+ (m_H - m_T)g + \frac{\ell_T - \ell_H}{2\ell} f_y + \frac{\ell_A}{\ell} f_x, \end{aligned} \tag{19}$$

while (13) is the same. From (4) and (5), $f_x = -F_x$ and $f_y = Mg - F_y$ are satisfied in the stationary state. Thus, the equilibrium point is obtained by solving two equations,

$$AL\sin(\bar{\theta} - \theta_f) + K_p(\theta_d - \bar{\theta}) + K_f\bar{\tau}_f = 0 \tag{20}$$

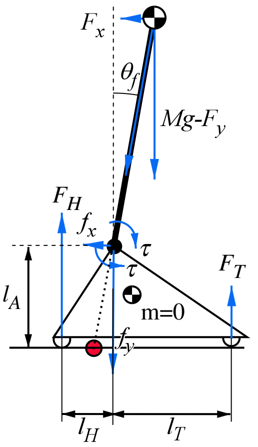


Figure 2: Stationary posture by Control 1.

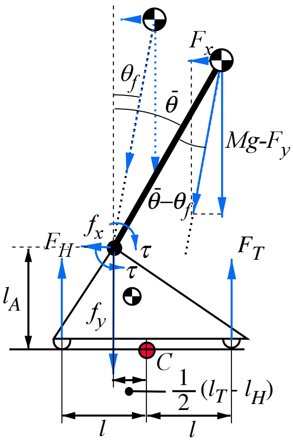


Figure 3: Stationary posture by Control 2.

$$\frac{1}{\ell}(K_p(\theta_d-\bar{\theta})+K_f\bar{\tau}_f)+(m_H-m_T)g$$
$$+\frac{\ell_T-\ell_H}{2\ell}(Mg-F_y)-\frac{\ell_A}{\ell}F_x=0 \qquad (21)$$

Firstly, let us discuss the stability of this equilibrium point. Linearizing the dynamics (13) and (19) around the equilibrium point, we obtain

$$I\ddot{\theta}=(AL\cos(\bar{\theta}-\theta_f)-K_p)\theta-K_d\dot{\theta}+K_f\tau_f \qquad (22)$$

$$\dot{\tau}_f=\frac{1}{\ell}(-K_d\dot{\theta}-K_p\theta+K_f\tau_f)$$
$$+\frac{ML}{\ell}\ddot{\theta}\left[-\frac{\ell_T-\ell_H}{2}\sin\bar{\theta}+\ell_A\cos\bar{\theta}\right] \qquad (23)$$

where, we used the following relations in the stationary state:

$$\bar{f}_x=ML\ddot{\theta}\cos\bar{\theta}-F_x \qquad (24)$$

$$\bar{f}_y=-ML\ddot{\theta}\sin\bar{\theta}+Mg-F_y \qquad (25)$$

Then, the characteristic equation becomes

$$\lambda^3+p_2\lambda^2+p_1\lambda+p_0=0 \qquad (26)$$

where

$$p_2=\frac{K_d\ell-K_f(I+f(\bar{\theta}))\ell}{I\ell}, \qquad (27)$$

$$p_1=\frac{K_p-AL\cos(\bar{\theta}-\theta_f)}{I}, \qquad (28)$$

$$p_0=\frac{K_fAL\cos(\bar{\theta}-\theta_f)}{I\ell} \qquad (29)$$

$$f(\bar{\theta})=\frac{ML}{\ell}\left[\frac{\ell_T-\ell_H}{2}\sin\bar{\theta}-\ell_A\cos\bar{\theta}\right]. \qquad (30)$$

Using Routh/Hurwitz method, the necessary and sufficient conditions that the equilibrium point becomes locally stable are.

$$K_d>(\frac{I}{\ell}+f(\bar{\theta}))K_f \qquad (31)$$

$$K_p>AL\cos(\bar{\theta}-\theta_f) \qquad (32)$$

$$K_f>0 \qquad (33)$$

$$(K_d-K_ff(\bar{\theta}))(K_p-AL\cos(\bar{\theta}-\theta_f))>\frac{I}{\ell}K_pK_f \qquad (34)$$

Next, let us discuss the stationary posture. From (20) and (21), we obtain the following equation in the stationary state:

$$AL\sin(\bar{\theta}-\theta_f)=(m_H-m_T)g\ell$$
$$+\frac{1}{2}(\ell_T-\ell_H)(Mg-F_y)-\ell_AF_x \qquad (35)$$

The left hand side represents torque to keep the body part steady at $\theta=\bar{\theta}$. On the other hand, the right hand side consists of three terms, which correspond to the moment around the point C caused by the foot weight, $Mg-F_y$ and F_x, respectively. Thus, the above equation implies that the body is inclined in order to cancel the moment of the foot around the point C which caused by the gravity force and external disturbance. Namely, the body inclination enhances the stability of the foot in the sense that the foot does not make rotation. The ankle joint torque, on the other hand, becomes

$$\tau=-(m_H-m_T)g\ell-\frac{1}{2}(\ell_T-\ell_H)(Mg-F_y)+\ell_AF_x \qquad (36)$$

Substituting it to (2) and (3), we obtain

$$F_T=F_H=\frac{1}{2}(m_H+m_T)g+\frac{1}{2}(Mg-F_y) \qquad (37)$$

This equation also indicates that the center of ground reaction forces is located at the center of the foot, i.e., the point C.

3. Human posture measurement

3.1 Objects

In the previous section, we have shown two control schemes, where either ankle joint output or stability is

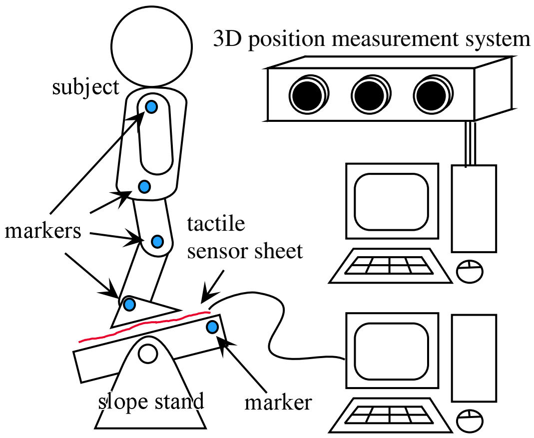


Figure 4: Measurement of human upright standing.

optimized at the stationary state. The Control 1 is the best from the aspect of energy efficiency, because only little torque is required for ankle joint to maintain the posture. However, the center of ground reaction forces tends to move into one side, and so the upright posture will be difficult to keep if the disturbance becomes large. The Control 2 is, on the other hand, the best from the aspect of the postural stability: The center of ground reaction force is controlled to the center of the foot by inclining the body part. Therefore, this posture is robust against perturbations, but the non-zero ankle joint torque is necessary to maintain this posture.

In this way, where the center of ground reaction forces should be controlled is an important problem in the motion planning of biped robot. Then, in the human upright posture, where does the center of ground reaction forces exist? In order to answer this question, we execute the following experiment.

3.2 Methods

In order to measure the distribution of ground reaction force, we used a tactile sensor system (F-scan, NITTA). This system contains a foot-shaped sensor sheet which consists of about 650 meshes every 5 mm in front-back and left-right direction. Calibrating the system, the forces are detected in 256 resolutions at each meshes. In addition, to record the posture during the experiments, we utilized a 3D position measurement system with three CCD cameras and target markers (OPTOTRACK, Northern Digital Systems). In the experiment, the spacial resolution of this position measurement system was set 1mm. The objects of our measurement were not motions but steady upright postures. Thus, the sampling time was set low 10 Hz, which is achievable in both the tactile and position sensor systems. The sensor sheet was put on the slope stand at the left foot position. To restrict the data analysis within the sagittal plane, it was aligned so as to face to the exact right direction against the measurement axis of the position measurement system.

Five subjects (all males, age 22-24 years) are examined. They were not told the object of this experiment before. During the measurement, they were instructed to stand up on the slope stand with whole sole on it, and keep the still posture in the relaxed form. In addition, focusing on the role of ankle joint, they also instructed not to bend knee or hip joint.

Five markers were used, four of which were attached to the ankle, knee, hip and shoulder joint position in the right hand side of subjects. And the rest one was set to the right hand side of slope stand so that it coincided the origin of the spatial coordinate of tactile sensor system. This 5-th marker provides the relative position of the center of the ground reaction force from the ankle joint.

Two kinds of experiments were performed. Firstly, the subjects were asked to stand on the slope stand with 0 deg. of the slope angle. This trial included three sets of measurement. In one set of measurement, the distribution of ground reaction forces as well as joint positions specified by markers were recorded during 10 seconds. Between two sets, the subjects were instructed to get off the slope stand in order to reset the creep property of the tactile sensor sheet. Next, the slope angle was changed to 5, 10 and 15 degree. This situation is equivalent to the one that the constant disturbance acts. The slope were made so that the toe side was higher than heel side, and its angle was adjusted using a protractor installed in the slope stand. Each trial included three sets of measurement as well. Fig. 4 illustrates the view of this experiment.

In the second experiments, the angle of slope stand was set to 0. Then, the subjects were instructed to stand on it with putting as large weight to the heel side as possible. In this trial, three sets of measurements were also performed in the same way. After the experiments, the subjects answered the question, 'Which were you tired, in this trial or previous one with 0 deg. of slope angle?'

3.3 Results

From the data of 3D position measurement system, knee and hip joint angles were calculated. By this analysis, we confirmed that the change of these angles were limited almost within 5 degrees for all subjects, which was smaller than that of the slope angle. From this results, we judged that the body part can be regarded as a single link model in Fig. 1.

Next, using the data of tactile sensor system, the position of the center of the ground reaction forces was calculated. Then, the forward deviations of the center of ground reaction forces from ankle joint position are computed using the information of the 5th maker position. Table 1 shows the time averages and standard deviations of them in each set. Conditions indicates the slope angle in the trials, but 'Heel-weighted' means the second experiment with putting the weight to the heel side. Foot center denotes a horizontal distance between the ankle joint and the center of foot, which is measured

Table 1: Forward deviation of the center of ground reaction forces from ankle joint position (mm).

Conditions		0 deg.	5 deg.	10 deg.	15 deg.	Heel-weighted	Foot center
Subject 1	1st	81 ± 1	73 ± 1	48 ± 3	49 ± 1	15 ± 3	77
	2nd	77 ± 1	73 ± 1	65 ± 2	49 ± 2	11 ± 3	
	3rd	71 ± 1	70 ± 2	55 ± 3	45 ± 1	8 ± 2	
	average	76 ± 4	72 ± 2	56 ± 7	47 ± 2	11 ± 4	
Subject 2	1st	70 ± 1	63 ± 1	50 ± 2	59 ± 3	19 ± 2	80
	2nd	70 ± 2	63 ± 4	45 ± 2	54 ± 3	29 ± 1	
	3rd	62 ± 1	60 ± 1	52 ± 1	36 ± 3	26 ± 2	
	average	68 ± 4	62 ± 3	49 ± 3	49 ± 10	25 ± 5	
Subject 3	1st	54 ± 2	83 ± 2	79 ± 2	78 ± 2	42 ± 3	75
	2nd	65 ± 2	68 ± 2	71 ± 2	76 ± 2	49 ± 1	
	3rd	75 ± 1	65 ± 2	83 ± 2	83 ± 2	31 ± 2	
	average	65 ± 9	72 ± 8	78 ± 6	79 ± 4	41 ± 8	
Subject 4	1st	50 ± 2	51 ± 1	42 ± 1	45 ± 1	51 ± 1	69
	2nd	60 ± 1	45 ± 1	50 ± 2	32 ± 2	24 ± 2	
	3rd	59 ± 2	43 ± 2	36 ± 1	42 ± 2	38 ± 1	
	average	56 ± 5	46 ± 4	43 ± 6	40 ± 6	38 ± 11	
Subject 5	1st	18 ± 2	8 ± 2	37 ± 1	51 ± 3	13 ± 3	75
	2nd	34 ± 3	43 ± 4	46 ± 2	52 ± 2	8 ± 2	
	3rd	33 ± 3	50 ± 1	43 ± 1	51 ± 3	1 ± 4	
	average	29 ± 8	33 ± 18	42 ± 4	52 ± 3	8 ± 6	

by a ruler.

The graph illustrated with the data normalized by each foot center value in Table 1 are shown in Fig. 5, where 0% means that the center of ground reaction force is just below the ankle joint, while 100% does at the center of the foot. Fig. 5(a) represents the changes with respect to the slope angle, while Fig. 5(b) shows the comparison between normal stance at the first experiment with 0 deg. of the slope angle and the one at the second experiment with putting the weight to the heel side. The positional changes of the center of ground reaction forces were different in subjects, but two tendencies were observed. The center of ground reaction forces moved backward in subject 1, 2 and 4, while forward in subject 1 and 5, when the slope angle became large. As a whole, it tended to be controlled closer to the center of foot than to the ankle joint (except subject 5).

In the second experiment, all subjects answered that they felt tired in the second experiment than in the first one with 0 deg. of the slope angle.

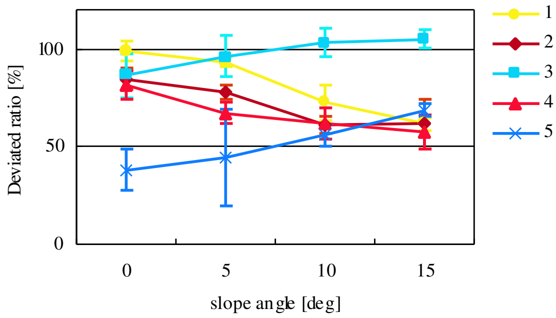
4. Discussion

If the human posture control were based on Control 1, the center of ground reaction forces would be controlled just below the ankle joint when the slope angle is set to 0 degree. However, in the experiment, the center of ground reaction forces was shifted to the toe side. Furthermore, the center of the ground reaction forces have existed at the back of the ankle joint when the toe side became higher than heel side by changing the slope angle. In the experiment, on the contrary, it still existed at the front of the ankle joint, and was closer to the center of the foot for almost all of the subjects (more than 50% in Fig. 5(a)). Even in the subject 5 whose center of ground reaction forces was the closet to the ankle joint when the slope angle was 0 deg., the deviated ration exceeded 50% in 5 and 10 deg. of the slope angle. These results indicate that the body is slightly inclined forward at the stationary state, and induces a negative answer to a hypothesis that the strategy of Control 1 would be adopted in the human posture control.

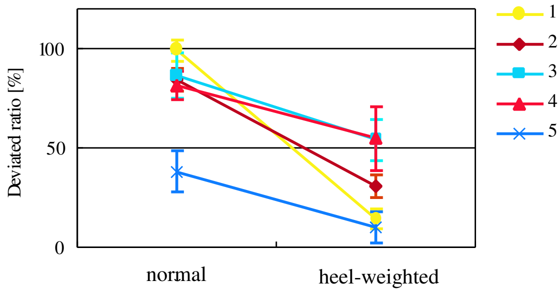
The answer of the subjects after the second experiment indicates that the Control 1 is not always efficient one in the human posture control. In the second experiments, the center of ground reaction forces was actually shifted backward as shown in Fig. 5(b). This fact theoretically means that the ankle joint output would becomes smaller than the first experiment. Nevertheless, all subjects felt more tired than before. One reason is that our model does not take into considerations the joint torque other than ankle joint. The skeletal structure or the rotational range of joints will be also important factors to argue the efficiency of posture maintenance. Humans may have a structure such that the postural maintenance becomes easy when the center of the ground reaction forces are controlled nearer to the center of the foot.

5. Conclusion

In this paper, we examined two upright posture control laws from the aspect of the stationary posture, its sta-



(a) Changes with slope angle.



(b) Changes beween normal and heel-weighted standing.

Figure 5: position of center of ground reaction forces from anke joint

bility and the required ankle joint torque using a simple two-link model. Two laws can directly control the center of the ground reaction forces, but differ in that where the center of the ground reaction forces are controlled at the level floor: just below the ankle joint, or the center of foot. The former is superior in the output evaluation of ankle joint, while the latter is in the postural balance. In order to clarify where the desired position of the center of the ground reaction forces should be set in the biped robot control, we measured the human biped posture for five subjects. From the experiments, it tended to be controlled rather to the center of foot, and it was difficult to conclude that the former control scheme is adopted in the human posture control. In addition, subjects answered that they felt more tired when they put the weight consciously at the heel, i.e., used the former control scheme. This fact implies that the two link model used in this paper is too simple to discuss the energy efficiency. So, as a future works, we should analyze it using multi-link model.

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