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The optimal object posture that minimizes contact forces in grasping

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Abstract

Because of the ill-posedness of grasping, one feasible method must be selected from possible strategies. Among many factors, this paper focuses on the posture of an object: which object direction is best when it is grasped. Then, the object is assumed to be held with three points where the contact forces can be generated in any directions. To evaluate the object posture, the norm of contact force vector consisting of the normal and tangential forces is selected. Consequently, the contact force becomes minimal when the center of mass of the grasped object and the centroid of the triangle composed by three contact points are aligned in the gravitational direction.

Keywords: Grasping; Object posture; Contact force; Minimization

1. Introduction

Grasp is a basic action for every task. Though it is a simple motion, its execution requires selective decisions of many factors: contact point placements, grasp mechanisms posture, magnitude or directions of contact forces and so on [1,2]. Despite some conditions or constraints, many degrees of freedom of grasp mechanisms bring multiple solutions on these factors.

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For the view point of the ill-posedness, lots of studies have been reported [3,4]. A standard framework for solving ill-posed problem is the optimization whose important attributes are evaluation functions, factors to optimize and solution algorithm. Although the norm of the contact force vector is a normal section, other evaluation functions are sometimes introduced, such as the force that compensates for the gravity [5], the balanced force with respect to the normalized external force [6,7] or output power for the grasping mechanisms [8,9]. The external force just before the object starts to move is also an effective candidate for the evaluation of power grasp [10,11]. On the other hand, the optimizing factors are widely discussed: the placement of the contact points [5–7], the posture of the grasping mechanisms [8,9] or the range of the contact point placement while keeping the force balance in three-dimensional space [12]. As for the methods for solving the optimization problems on the grasp, linear programmings [13,14], quadratic programmings [15], artificial neural networks [16,17] and fuzzy logic [18] have been applied.

Among many optimizing factors, the posture of the grasped object is focused on in this paper. The posture here means the attitude of the grasped object in the task coordinate frame—in other words, spacial relation with respect to the gravity. The posture of the grasped object is usually unconstrained except, e.g., when delivering a glass of water. Thus, it is worth discussing from the efficiency point of view. However, this issue was not directly treated in the previous works mentioned above besides our studies [19].

This paper is organized as follows: In Section 2, we describe the problem to consider clearly under some assumptions for mathematical calculations. Next, the problem is formulated as a non-linear optimization problem in Section 3. This problem is solved in a numerical manner at first in Section 4 with examples. Then, the result is extended by solving the problem in an analytical manner in Section 5. Finally, we conclude this paper in Section 6.

2. Problem and assumptions

This paper aims at clarifying which object posture is best in the grasp. As an evaluation of the grasp, the magnitude of the contact forces is focused on here: small contact forces result in the efficient grasp as well as the avoidance of breaking the grasped object. In addition, it will provides a pure evaluation of the grasping configurations regardless of the specification of the grasping mechanisms. Then, the grasping mechanism is implicitly assumed to move to any direction in the space so as to adjust the object posture, and to generate the contact force to any directions at each contact points.

In summary, the problem is described as:

- Raise up the object from the upper side with three assigned contact points, i.e., the centroid of the contact-point triangle is located at the higher position than the object's center of mass. Then, find the optimal posture of the object in the sense that the contact forces become minimal.

Under the assumptions that

- The object is rigid.
- The object's shape is convex, and smooth at the contact points.

- At the contact points, the contact force is generatedin any direction and in any amount.

The last assumption implies that the object would be held using something like a fixture or magnetic forces. Regarding to the contact points, the grasp with the smallest number, i.e., three [20], of them is treated from the standpoint that extra mechanisms require unnecessary costs.

The above problem could be solved by use of the manipulability [21], but the spacial relation of the optimal solution is not sufficiently investigated. The focus of this paper is placed on clarifying the physical meaning of the optimal posture.

3. Formulation

Grasping tasks are sometimes described in the task coordinate frame, an orthogonal coordinate frame whose origin is fixed on a point in the task space of the grasp and one of three orthogonal axes of which is sometimes selected to be parallel to the gravitational direction. The posture in the task coordinate frame is represented as the relative direction of the gravity in the object coordinate frame O-XYZ whose origin is set to the center of this object' mass. Two parameters, ϕ and θ, express this direction: ϕ $(0 \leq \phi \leq \pi/2)$ is an angle between the gravitational direction and the negative direction of Z-axis, and θ is the azimuthal angle of the gravitational direction measured from the positive direction of the X-axis in the X–Y plane, as illustrated in Fig. 1.

The shape of an object is expressed by a convex function $f(x, y, z) = 0$ defined in the object coordinate frame. Three points $\boldsymbol{p}_i = (x_i, y_i, z_i)^T$ $(i = 1, 2, 3)$ are given on the object surface $f(x, y, z) = 0$ as contact points. They never align on the same straight line.

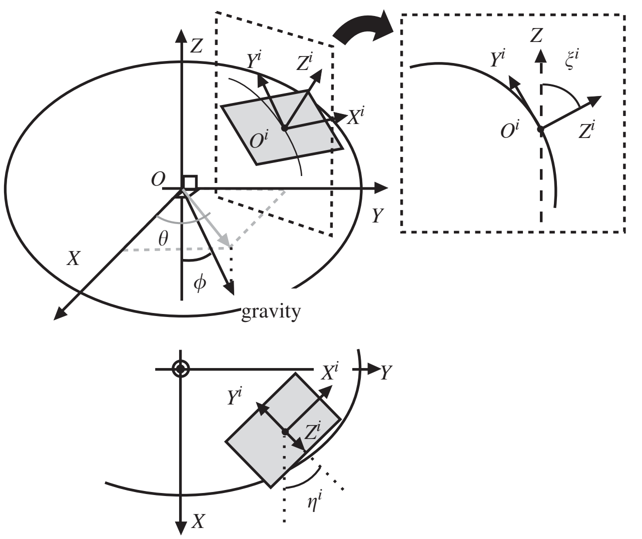

Fig. 1. Object coordinate frame and contact-point coordinate frame.

The contact-point coordinate frame O_i-$X_iY_iZ_i$ $(i = 1, 2, 3)$ is introduced to express the contact force as shown in Fig. 1. The origin of this coordinate frame is set to each contact point and each axis is defined as follows: the normal direction at the contact point $\boldsymbol{p}_i$, $(\partial f/\partial x, \partial f/\partial y, \partial f/\partial z)_{\boldsymbol{p}_i}$ is selected as the Z_i-axis, and the Y_i-axis is defined to be orthogonal to both the Z_i and the Z-axes. The X_i-axis is defined so that O_i-$X_iY_iZ_i$ become the right-handed coordinate frame. In the contact-point coordinate frame, the contact force $\boldsymbol{F_i} = [F_{ix}, F_{iy}, F_{iz}]^T$ is defined at each contact point. F_{iz} is the normal force whereas F_{ix} and F_{iy} describe the tangential force. The evaluation function is defined using $\boldsymbol{F} = [\boldsymbol{F}_1^T, \boldsymbol{F}_2^T, \boldsymbol{F}_3^T]^T$ as

$$V = \|\boldsymbol{F}\|^2 = \sum_{i=1}^{3}(F_{ix}^2 + F_{iy}^2 + F_{iz}^2) \tag{1}$$

in the contact-point coordinate frame.

Though the contact force expression $\boldsymbol{F_i}$ in the contact-point coordinate frame provides a simple description of the evaluation function as the above, it is convenient to describe the force balance equation in the object coordinate frame. Thus, $\boldsymbol{F_i}$ is transformed to $\boldsymbol{f}_i$, the expression in the object coordinate frame, using a transform matrix T_i.

$$\boldsymbol{f}_i = T_i\boldsymbol{F}_i \tag{2}$$

Here, let ξ_i to be the angle made at the meeting of the Z-axis and the Z_i-axis, as well as η_i to be the azimuthal angle made at the meeting of the Z_i-axis and the X-axis in the X–Y plane, as shown in Fig. 1. Then, T_i is obtained from the combined transformation, i.e., the ξ_i-rotation around X_i-axis, $rot(X_i, \xi)$, before the $(\eta_i + \pi/2)$-rotation around Z_i-axis, $rot(Z_i, \eta_i + \pi/2)$:

$$\begin{aligned} T_i &= rot(Z_i, \eta_i + \pi/2) \cdot rot(X_i, \xi) \\ &= \begin{bmatrix} \cos(\eta_i + \pi/2) & -\sin(\eta_i + \pi/2) & 0 \\ \sin(\eta_i + \pi/2) & \cos(\eta_i + \pi/2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\xi_i & -\sin\xi_i \\ 0 & \sin\xi_i & \cos\xi_i \end{bmatrix} \\ &= \begin{bmatrix} -\sin\eta_i & -\cos\eta_i\cos\xi_i & \cos\eta_i\sin\xi_i \\ \cos\eta_i & -\sin\eta_i\cos\xi_i & \sin\eta_i\sin\xi_i \\ 0 & \sin\xi_i & \cos\xi_i \end{bmatrix} \end{aligned} \tag{3}$$

Now, the force balance equation is given at the object coordinate frame as follows:

$$L\boldsymbol{f} = \boldsymbol{M} \tag{4}$$

Here, $L \in \boldsymbol{R}^{6\times 9}$ is a grasp matrix given by

$$L = \begin{bmatrix} I_3 & I_3 & I_3 \\ R_1 & R_2 & R_3 \end{bmatrix} \tag{5}$$

where $I_3 \in \boldsymbol{R}^{3\times 3}$ is a unit matrix and $R_i \in \boldsymbol{R}^{3\times 3}$ is a skew-symmetrical matrix determined by the position of the contact point:

$$R_i = \begin{bmatrix} 0 & -z_i & y_i \\ z_i & 0 & -x_i \\ -y_i & x_i & 0 \end{bmatrix} \tag{6}$$

$\boldsymbol{f} = [\boldsymbol{f}_1^T, \boldsymbol{f}_2^T, \boldsymbol{f}_3^T]^T$ is a contact force vector. $\boldsymbol{M}$ is a vector given by

$$\boldsymbol{M} = [M_x \;\; M_y \;\; M_z \;\; 0 \;\; 0 \;\; 0]^T \tag{7}$$

$$M_x = Mg\cos\theta\sin\phi \tag{8}$$

$$M_y = Mg\sin\theta\sin\phi \tag{9}$$

$$M_z = Mg\cos\phi \tag{10}$$

that denoting the direction of the gravitational force. Here, M is the mass of the object and g is the gravity acceleration.

The purpose here is to find the solution of the force balance equation (4) that minimize the evaluation function (1). Note that the solution of the force balance equation depends on both ϕ and θ. Namely, this is an optimization problem of the contact force vector by selecting the object posture ϕ and θ, i.e., the angles θ and ϕ are the variables to optimize.

4. Numerical analysis by case studies

4.1. Introduction of numerical analysis

The solution of Eq. (4) is represented with $L^\dagger$, the pseudo-inverse matrix of L [22] as

$$\boldsymbol{f} = L^\dagger \boldsymbol{M} + (I_9 - L^\dagger L)\boldsymbol{p} \tag{11}$$

where $I_9 \in \boldsymbol{R}^{9\times 9}$ is a unit matrix, and $\boldsymbol{p} \in \boldsymbol{R}^9$ is an arbitrary vector. The second term can be expressed using the unit vectors $\boldsymbol{e}_{ij} \in \boldsymbol{R}^3$ $(i,j = 1,2,3)$ given as

$$\boldsymbol{e}_{ij} = \frac{\boldsymbol{p}_i - \boldsymbol{p}_j}{\|\boldsymbol{p}_i - \boldsymbol{p}_j\|} \tag{12}$$

Namely,

$$(I_9 - L^\dagger L)\boldsymbol{p} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{e}_{13} & \boldsymbol{e}_{12} \\ \boldsymbol{e}_{23} & \boldsymbol{0} & \boldsymbol{e}_{21} \\ \boldsymbol{e}_{32} & \boldsymbol{e}_{31} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = L_N \boldsymbol{\alpha} \tag{13}$$

where each α_i correspond to the magnitude of the internal forces [23]. These internal forces do not affect the object motions because they cancel each other, while the first term of Eq. (11) actually balance the gravity effect. Accordingly, the solutions of Eq. (4) are represented by

$$\boldsymbol{f} = (L^\dagger \boldsymbol{M} + L_N \boldsymbol{\alpha}) \tag{14}$$

Among these solutions, the optimal one that minimizes the evaluation function (1) must be selected. From Eq. (2), $\boldsymbol{F}$ is given by

$$\boldsymbol{F} = T^{-1}(L^{\dagger}\boldsymbol{M} + L_N\boldsymbol{\alpha}) \tag{15}$$

where

$$T = diag[T_1, T_2, T_3] \tag{16}$$

Thanks to the orthogonality between $L^{\dagger}$ and L_N as well as $T^{-1} = T^T$, the evaluation function becomes

$$V = \boldsymbol{M}^T(LL^T)^{-1}\boldsymbol{M} + \|\boldsymbol{\alpha}\|^2 \tag{17}$$

Obviously, the smaller the norm of $\boldsymbol{\alpha}$ is, the better the evaluation function becomes. This is a reason why $\boldsymbol{\alpha}$ is assumed to be zero, which enables the following analyses to be restricted to finding the optimal angle ϕ and θ regardless of the magnitude of $\boldsymbol{\alpha}$.

However, the straightforward analysis is difficult due to the complexity of high dimensional calculations. Thus, some case studies are firstly examined to make an induction of a general conclusion.

4.2. Contact points constructing an equilateral triangle

Consider the case where the contact points are given as follows:

$$\boldsymbol{p}_1 = (6 \ \ -2\sqrt{3} \ \ 4)^T \tag{18}$$

$$\boldsymbol{p}_2 = (-6 \ \ -2\sqrt{3} \ \ 4)^T \tag{19}$$

$$\boldsymbol{p}_3 = (0 \ \ 4\sqrt{3} \ \ 4)^T \tag{20}$$

Note that these three points construct an equilateral triangle (Fig. 2a). Then, the evaluation function becomes

$$V = \frac{(Mg)^2}{9}(5 - 2\cos^2\phi) \tag{21}$$

Thus, the angles that minimize this evaluation function, i.e., the solution of $\partial V/\partial\phi = 0$ is

$$\phi = 0 \tag{22}$$

This result implies that the axis of gravitational force from the center of object's mass goes through one of the following points: incenter, circumcenter, orthocenter or centroid of the equilateral triangle.

4.3. Contact points constructing an isosceles right triangle

Consider the case where the contact points are given as follows:

$$\boldsymbol{p}_1 = (4\sqrt{3} \ \ 0 \ \ 4)^T \tag{23}$$

$$\boldsymbol{p}_2 = (-4\sqrt{3} \ \ 0 \ \ 4)^T \tag{24}$$

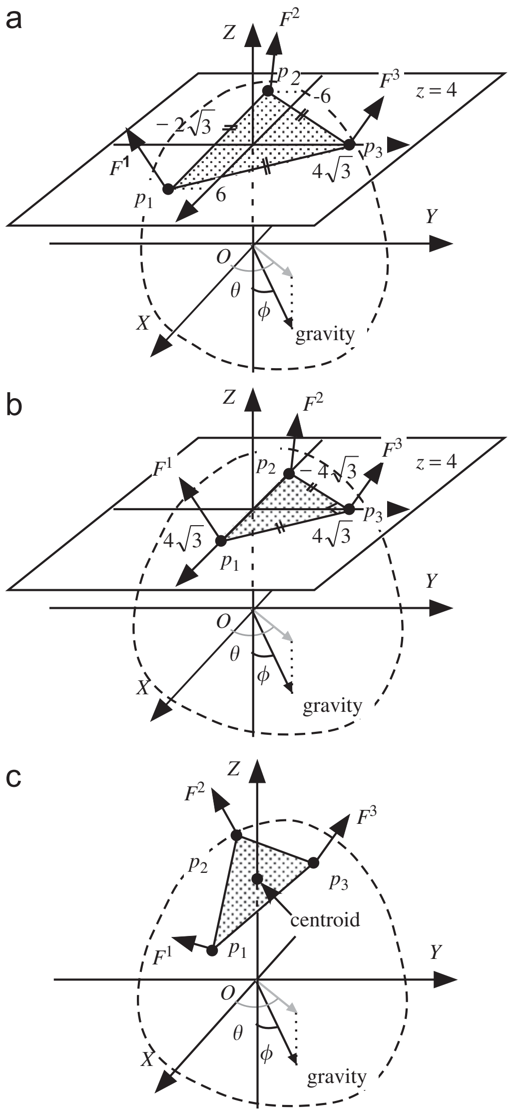

Fig. 2. Examples of contact-point triangle: (a) equilateral triangle, (b) right isosceles triangle, and (c) general case.

$$\boldsymbol{p}_3 = (0 \;\; 4\sqrt{3} \;\; 4)^T \tag{25}$$

Note that these three points construct an isosceles right triangle (Fig. 2b). Then, the evaluation function becomes

$$V = \frac{(Mg)^2}{24}(13\cos^2\theta\sin^2\phi + 20\sin^2\theta\sin^2\phi - 8\sqrt{3}\,\sin\theta\,\sin\phi\cos\phi + 12\cos^2\phi) \tag{26}$$

For minimization, put the derivative of V to zero as

$$\frac{\partial V}{\partial \theta} = \frac{(Mg)^2}{24}(14\cos\theta\sin\theta\sin^2\phi - 8\sqrt{3}\cos\theta\sin\phi\cos\phi) = 0 \tag{27}$$

$$\frac{\partial V}{\partial \phi} = -\frac{(Mg)^2}{12}(7\cos^2\theta\sin\phi\cos\phi - 8\cos\phi\sin\phi + 8\sqrt{3}\sin\theta\cos^2\phi - 4\sqrt{3}\sin\theta) = 0 \tag{28}$$

Solving the above two simultaneous equations, we obtain the solution

$$(\theta, \phi) = \left(-\frac{\pi}{2}, \frac{\pi}{6}\right) \tag{29}$$

This result implies that the axis of gravitational force from the center of object's mass goes through the centroid of the isosceles right triangle.

5. Mathematical analysis and result

Two case studies in Sections 4.2 and 4.3 provide a suggestive result: if the line from center of object's mass to the direction of the gravitational force passes the centroid of the triangle composed by three contact points, then the norm of the contact force vector becomes minimal. In this section, we aim at generalizing this numerically induced result for arbitrary placements of contact points.

At first, the object coordinate frame is reset so that the Z-axis goes through the centroid of the contact-point triangle (Fig. 2c). The contact points are re-defined in the new coordinate frame:

$$\boldsymbol{p}_1 = (x_1 \;\; y_1 \;\; z_1)^T \tag{30}$$

$$\boldsymbol{p}_2 = (x_2 \;\; y_2 \;\; z_2)^T \tag{31}$$

$$\boldsymbol{p}_3 = (x_3 \;\; y_3 \;\; z_3)^T \tag{32}$$

Here, the following relations hold:

$$x_1 + x_2 + x_3 = 0 \tag{33}$$

$$y_1 + y_2 + y_3 = 0 \tag{34}$$

$$z_1 + z_2 + z_3 > 0 \tag{35}$$

Eq. (35) ensures that the object is hold from the upper side.

Then, the matrix LL^T in Eq. (17) is given as

$$LL^T = \begin{bmatrix} 3 & 0 & 0 & 0 & Z & 0 \\ 0 & 3 & 0 & -Z & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & -Z & 0 & \alpha & \lambda & \kappa \\ Z & 0 & 0 & \lambda & \beta & \nu \\ 0 & 0 & 0 & \kappa & \nu & \gamma \end{bmatrix} \tag{36}$$

where

$$Z = z_1 + z_2 + z_3 \tag{37}$$

$$\alpha = y_1^2 + y_2^2 + y_3^2 + z_1^2 + z_2^2 + z_3^2 \tag{38}$$

$$\beta = z_1^2 + z_2^2 + z_3^2 + x_1^2 + x_2^2 + x_3^2 \tag{39}$$

$$\gamma = x_1^2 + x_2^2 + x_3^2 + y_1^2 + y_2^2 + y_3^2 \tag{40}$$

$$\lambda = -x_1 y_1 - x_2 y_2 - x_3 y_3 \tag{41}$$

$$\kappa = -z_1 x_1 - z_2 x_2 - z_3 x_3 \tag{42}$$

$$v = -y_1 z_1 - y_2 z_2 - z_3 y_3 \tag{43}$$

Next, the evaluation function becomes

$$V = \frac{(Mg)^2}{E_0}(E_1 \cos^2\theta \sin^2\phi + E_2 \cos^2\phi + E_3 \cos\theta \sin\theta \sin^2\phi + E_4) \tag{44}$$

Here, E_k $(k = 0, \ldots, 4)$ is a constant that is determined by x_i, y_i, z_i $(i = 1, 2, 3)$ (see Appendix). From the conditions $\partial V/\partial\phi = 0$ and $\partial V/\partial\theta = 0$, the following equations are obtained:

$$\frac{(Mg)^2 \sin^2\phi}{E_0}(-E_1 \sin 2\theta + E_3 \cos 2\theta) = 0 \tag{45}$$

$$\frac{(Mg)^2 \sin\phi \cos\phi}{E_0}(E_1 \cos 2\theta + E_3 \sin 2\theta + E_1 - 2E_2) = 0 \tag{46}$$

There are two possible cases, i.e., $\sin\phi = 0$ or the case where the following two equations simultaneously hold:

$$-E_1 \sin 2\theta + E_3 \cos 2\theta = 0 \tag{47}$$

$$E_1 \cos 2\theta + E_3 \sin 2\theta + E_1 - 2E_2 = 0 \tag{48}$$

The latter case, however, has no solutions since $\sin 2\theta$ and $\cos 2\theta$ must satisfy the third equation:

$$\sin^2 2\theta + \cos^2 2\theta = 1 \tag{49}$$

Thus, the solution of two simultaneous Eqs. (45) and (46) is

$$\phi = 0 \quad (\theta \text{ is arbitrary}) \tag{50}$$

When $\phi = 0$, the direction of gravitational force coincides with the Z-axis that goes through the centroid of the contact-point triangle.

The result of the above analysis is summarized to the following theorem.

Theorem. *Consider that a rigid and convex object is hold with three contact points. If contact force can be generated in any direction at each contact point, then the squared sum of the contact forces takes minimum at the object posture where the centroid of contact-point triangle and the center of mass of the object are aligned in the gravitational direction.*

6. Conclusion

In this paper, the posture of the grasped object that minimizes the contact forces is discussed when the object is held up with three contact points given in advance. The squared norm of the contact force vector whose component is the normal or tangential force at each contact point is selected as the evaluation function. Assuming that the contact force acts in any direction, the following results are obtained with respect to convex and rigid objects:

- The posture in which the centroid of the triangle composed of three contact points and the center of the object's mass are aligned in the gravitational direction becomes optimal.

The contribution of this paper is to describe the spacial relation of the optimal object's posture from the physical point of view. These results are not different from our intuition. Though the result may seem obvious, it is meaningful to have ensured the rightness of our intuition mathematically.

The above result is derived neglecting the internal forces since zero internal forces make contact forces smallest. However, usual grasp by hand utilizes the friction forces, where the effect of internal forces increases the normal forces not to slip out the grasped object. As future works, we will examine the grasping action with friction and next consider the manipulation of an object with friction.

Appendix A. Derivation of (44)

Let $\Lambda = (LL^T)^{-1}$, and Λ_{ij} denotes the i-column j-row element of the matrix Λ. Because Λ is symmetrical matrix and $\Lambda_{13} = \Lambda_{23} = 0$,

$$\begin{aligned} V &= \boldsymbol{M}^T(LL^T)^{-1}\boldsymbol{M} \\ &= (Mg)^2(\Lambda_{11}\cos^2\theta\sin^2\phi + 2\Lambda_{12}\cos\theta\,\sin\theta\sin^2\phi + \Lambda_{22}\sin^2\theta\sin^2\phi + \Lambda_{33}\cos^2\phi) \\ &= (Mg)^2[(\Lambda_{11} - \Lambda_{22})\cos^2\theta\sin^2\phi + (\Lambda_{33} - \Lambda_{22})\cos^2\phi \\ &\quad + 2\Lambda_{12}\cos\theta\sin\,\theta\sin^2\phi + \Lambda_{22}] \end{aligned} \tag{A.1}$$

Thus, let $E_0 = det(LL^T) \neq 0$, then $E_1 = (\Lambda_{11} - \Lambda_{22})E_0$, $E_2 = (\Lambda_{33} - \Lambda_{22})E_0$, $E_3 = 2\Lambda_{12}E_0$ and $E_4 = \Lambda_{22}E_0$.

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