A Pictorial Pattern Recognition Based on an Associative Memory by Use of the Reaction-Diffusion Equation

Satoshi ITO[1)] Hideo YUASA[2),1)] Koji ITO[3),1)] Masami ITO[4),1)]

[1)]Bio-Mimetic Control Reserch Center, the Institute of Physical and Chemical Research (RIKEN), Nagoya, 456, Japan

[2)]School of Engineering, Nagoya University, Nagoya, 464-01, Japan

[3)]Department of Information and Computer Science,Toyohashi University of Technology, Toyohasi, 441, Japan

[4)]School of Engineering, Chubu University, Kasugai, 487, Japan

Abstract

Visual information process in the biological systems is performed simultaneously and in parallel at each neural unit. That is, a global task with no supervisor is achieved by the interaction among a lot of subsystems. In this sense, the visual process has a share of the common functional structure with 'Autonomous Decentralized Systems' which is attracted much attention in the fields of the system engineering.

In the present paper, we attempt to design a pictorial pattern recognition system based on the concept of the autonomous decentralized system. The local interaction among subsystems is described by reaction-diffusion equations, in which each subsystem utilizes only the neighboring information around itself.

1 Introduction

The scientific technology has made our labor easy and able to be done efficiently. Consequently, machines with developed technologies have taken the places of the human at some aspects. But a number of machines are still inferior to the human in adaptability and flexibility. In these days, it is much required to study the mechanism or methodology which realizes adaptive and flexible mechanisms or machines. *'Autonomous Decentralized Systems'* is one of the promising candidates referring to the adaptation to varying or unpredictable circumstances. The autonomous decentralized system is a system in which a functional order over the entire system is generated by cooperative interactions among its subsystems, each of which holds its own autonomy to be able to control a part of the states of the system.

It is well known that biological system possesses such an autonomous decentralized characteristic. In fact, it is capable of self-organizing various kinds of functional order by autonomous coordination of many system elements. For example, it is known that in human memory, memorization and recall are caused by cooperation of many neurons. Since many neural circuits can change the weights of the synapses through the learning, great number of memory patterns corresponding to the various purposes are able to be formed flexibly. Further it is able to consider that artificial neural network is one of the autonomous decentralized structure. Then it is important to define explicitly the relationship between the interaction among subsystems and the order of the entire system.

As a model of human memory, H.Haken proposed the way to design the auto-associative memory (see section 2.1) by synergetic computing, where the recalling patterns are described by the differential equations and each stable state of the system corresponds to the memorized picture respectively ([2], [4], [5], [6]). However the pictorial pattern data are represented as a vector data of pixel values, which are picked up in order from the picture and arrayed in one row. Since the two-dimensional connections have been lost, the recognition process requires the global information. Many other methods are proposed, e.g., an associative memory by use of neural network [8], an orthogonal projection ([9], [10]), ordinary serial processes with feature abstract and so on. However all of them can not be realized without utilizing the global information over the pictures.

The present paper proposes a the model of the human memory based on the concept of the autonomous decentralized system. Especially, we aim to realize the recalling process merely by the local interaction among the subsystems. In our method, the interactions is performed by the diffusion which is done in parallel.

0-8186-7087-8/95 $4.00 © 1995 IEEE

Therefore, the local operation makes the recognition rapidly similar to the visual process in the brain.

2 Pictorial Pattern Recognition System

2.1 Fundamental Ideas

In the present paper, it is shown how to design the pictorial pattern recognition system in view of the autonomous decentralized system. The recognition system should be composed of many subsystems, and the interaction among subsystems regulate the global order of whole system. At first, we divide the memorized or inputted pictures into many parts. Then every part (pixel unit) behaves of themselves and interacts mutually. Finally the global order is formed spontaneously, which corresponds to the recall of the memorized picture.

Here we adopt the method of auto-associative memory [2], which is explained as follows. Let's assume that a couple of data (x_1, y_1), ..., (x_n, y_n) are stored in the memory. The associative memory is the system which outputs the data y_i for the input x_i. The associative memory is called *'auto-associative memory'* in the case that the output data are equal to the input data, i.e., $y_i = x_i$. The auto-associative memory outputs the most similar data to the input pattern in the memory.

Now first let's use the ideas of the orthogonal projection.

When the system receives a test pattern (input picture) q, it is decomposed to the component of each prototype pattern (memorized picture) v_k $(k = 1,\ldots,M$; M is the number of the memorized pictures) and their residue w such as

$$q = \sum_{i=1}^{M} a_i v_i + w. \tag{1}$$

Here, it should be noted that q, $v_k(k = 1, \ldots, M)$ and w may be either vector or function generally. However, throughout this section, q and v_k are defined as scalar functions, for example $q(x, y)$ or $v_i(x, y)$, which denotes the gray level of the pixel located at the coordinate (x, y) on the face of the picture. For the sake of simplicity, we assume that $q(x, y)$ and $v_i(x, y)$ are all C^∞. The a_k denotes the weight of the prototype patterns which the test pattern includes.

To begin with, we construct the system which outputs only the projected component on the subspace Π spanned by the memorized patterns, i.e.

$$\tilde{q}(x, y) = \sum_{i=1}^{M} a_i v_i(x, y). \tag{2}$$

In order to output $\tilde{q}(x, y)$, the system must know the weight a_k. These a_k can be calculated by use of the adjoint function v_i^* such as

$$a_i = \iint_\Omega v_i^* q(x, y) dx dy, \tag{3}$$

where Ω denotes the whole picture ($\|\Omega\| < \infty$). The adjoint function v_i^* satisfies

$$v_i^* = \sum_{j=1}^{M} c_j v_j(x, y), \tag{4}$$

and

$$\iint_\Omega v_i^* v_j(x, y) dx dy = \delta_{ij}, \tag{5}$$

where δ_{ij} is Kronecker's delta.

Eq.(4) means that the adjoint function v_i^* is on the function space Π spanned by the prototype functions v_i. Accordingly the component of the test function except the function space Π can be deleted. Eq.(5) means that every adjoint function v_i^* orthogonizes to each $v_j (j \neq i)$ and the product with v_i is unit.

Considering from the view of the autonomous decentralization, the system should not use the global information such as the function values of all the coordinates in the picture. When we calculate eq.(3) directly, we must utilize the global information in the form of function values. For the use of the local information only, we introduce the diffusion which makes each pixel value converge to the correct integral value.

Accordingly, the problem turns as follows. When the function f is defined on the bounded set Ω, we may find a procedure to calculate the following equation

$$f_c = \int_\Omega f d\sigma \tag{6}$$

by the local operation. If we put $f = v_i^*(x, y)q(x, y)$, $d\sigma = dxdy$, eq.(6) changes to eq.(3). For the sake of simplicity, first we assume that the function f has only one argument.

Here, we consider how to calculate the integral in the right hand side of eq.(6) by the diffusion on the picture plane. The integral calculation by the diffusion will become possible by introducing some local potential function u_i whose minimum corresponds to the correct integral value. We assign the local potential function as

$$u_i = \lambda \left[\frac{f(x_i) - \hat{f}_c^i}{\Delta} \right]^2, \tag{7}$$

where $\hat{f}_c^i$ is the estimate of the function value f_c^i at the coordinate x_i, Δ denotes a pixel width between x_i (x_{i-1}) and x_{i+1} (x_i). As we make the function f evolved temporally at each pixel according to this potential function, we obtain

$$\begin{aligned} \frac{\partial f(x_i)}{\partial t} &= -\frac{\partial u_i}{\partial f(x_i)} \\ &= \frac{4}{9}\lambda_i \frac{1}{\Delta}\Big(\frac{f(x_{i+1}) - f(x_i)}{\Delta} \\ &\quad -\frac{f(x_i) - f(x_{i-1})}{\Delta}\Big), \end{aligned} \tag{8}$$

where, λ_i is a parameter which determines the rate that $f(x_i)$ converges to f_c. As Δ goes to 0, eq.(8) can be written as the diffusion equation.

$$\frac{\partial f(x_i)}{\partial t} = \lambda \nabla_x^2 f(x, y), \tag{9}$$

where λ is a diffusion constant.

Before putting use to the diffusion on the picture plane, we must initialize the value f_i at each coordinate, which corresponds to the local estimate of the integrated value of eq.(6). That is, in each coordinate the approximated value $\hat{f}_c^i$ is locally estimated by use of the information of neighboring pixels. Here we give as follows.

$$\hat{f}_c^i = \frac{1}{3} \sum_{j=i-1}^{i+1} f(x_i). \tag{10}$$

Eq.(10) means that the estimate of the integral value is given by the average of the pixel values around. It is easy to understand that eq.(10) is appropriate, because the average of the area approximated by the squares is given by eq.(10).

In the initial state each estimate $\hat{f}_c^i$ has a different value in each coordinate. But after the evolution in time according to eq.(9), they become to take the same values f_c (see Fig.1), which is obvious from the fact that the potential function eq.(7) takes the minimum at f_c.

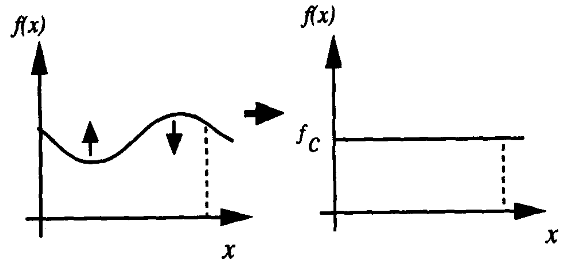


Fig. 1 Averaging by diffusion.

Finally we must mention about the boundary condition of eq.(9). To calculate the integral correctly, we have to give the boundary condition to eq.(9) such that the area surrounded by function f and x-axes can not change. For the simplicity we regard the boundary in the left and right side of the area as the same throughout this article, which keeps the area constant.

It is easy to extend eq.(9) to two-dimensional case. In that case, it is possible to delete the boundary condition by considering the picture as two-dimensional torus virtually.

The system outputs the picture using the values calculated according to eq.(9) as

$$q(x, y; t) = \sum_{i=1}^{M} a_i(x, y; t) v_i(x, y). \tag{11}$$

Since every $a_i(x, y; t)$ converges to the value a_i given by eq.(3), $q(x, y; t)$ also converges to $\tilde{q}$ of eq.(2).

It is important that eq.(9) is calculated only by the local operation.

2.2 The System with Reaction-Diffusion Equation

There are some problems in the algorithm mentioned in the previous section. For example,

- It is necessary to compute the adjoint function of each memorized function (picture) so as to set the initial value at every location on the picture. Then we have to know all the functions first to calculate the adjoint functions.

- The output pattern does not include the component except the function space Π spanned by the memorized pictures. Then it can converge to the function space Π, but cannot converge to one of the memorized pattern. It means that this system doesn't have the ability to get rid of the noises in the function space spanned by the memorized pictures.

It should be tried to design such a system that can solve those problems. For this purpose, we might add the reaction term to the diffusion equation. Then we obtain the reaction-diffusion equations that describe the dynamic property of the system.

2.2.1 Active Elements and Phase Initialization

Here we adopt an idea of *'active elements'* to explain our ideas easily. The active element is assigned to each pixel on all the memorized pictures by one-to-one. The elements are arranged by the same order as the pixels to which the elements are assigned, and then the plane made from the elements is obtained. It is trivial that the number of the planes is equal to that of the memorized pictures. To refer to these planes, the term *'element plane'* can be used. Further, the term *'group'* has to be defined. The term *'group'* represents a set of the elements that is located at the same coordinate on each element plane (see Fig.2).

It is assumed that the active element has its own dynamic property. To put it more concretely, these elements have two stable states. The value in the stable state are supposing to be 0 and 1. In addition, the state of element is represented by the value from 0 to 1. The value corresponding to the state of element is called here the term *'phase'*. When the test and memorized picture are compared in microscopic, i.e., at the pixel size level, the phase is equivalent to the index that indicates a kind of the distance between the test picture and the memorized picture to which each element is assigned. If two pictures are the same one, the phase will take the value 1. On the other hand, if it is concluded that two pictures are not, the phase will take the value close to 0.

What has to be noticed here is that when a test pattern is inputted, the initial value of the phase are decided uniquely from the comparison at the pixel level. Then the phase changes dynamically to settle in the stable state suitable for the inputted pattern. (The latter phase dynamics is taken up in the next section.) As for the former, the phase is initialized by the following equation,

$$a_k(\xi,0) = f(p(\xi), v_k(\xi)) = \frac{c}{[p(\xi)-v_k(\xi)]^\alpha + c}, \qquad (12)$$

where, $\xi = (x,y)$ denotes the coordinate of the pixel on the picture, t denotes the time, $a_k(\xi,t)$ denotes the phase of the element at the time t, which is assigned to the pixel located at the coordinate ξ on the k-th picture. The parameters α and c are constant. In this paper, these α and c are set to 4 and 400 respectively. The graph of eq.(12) is shown in Fig. 3.

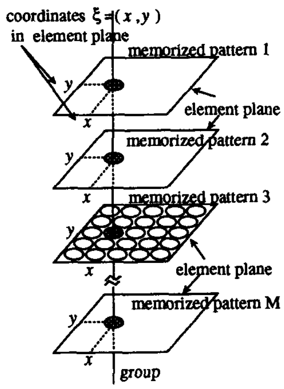


Fig. 2 Element planes and group

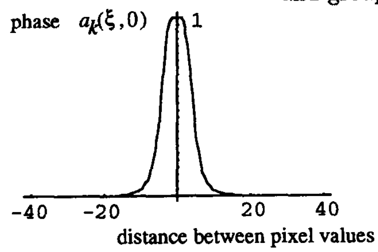


Fig. 3 The function that initialize the phase of the element.

Eq.(12) is equivalent to the spatial low-pass filter. The constant c corresponds to the parameter that determines the spatial range of the filter. If the difference between pixels is large, the phase will be set to 0. Thus the characteristics of the low-pass filter gives the system robustness against the high-frequency noises.

2.2.2 Phase Dynamics

As to the dynamic property of the global set of active elements, the following two macroscopic factors are necessary to achieve the pattern recognition.

(1) After evolution in time, all elements in the same element plane take the same phase value.

(2) In every group, only one element takes the phase value 1 and all the others take the phase value 0 at the stable state.

To make the system settle in the stable states where the above two conditions are satisfied (Fig. 4), the dynamics of the active element is defined by the reaction-diffusion equation in the following way.

$$\frac{\partial a_k(\xi,t)}{\partial t} = -\frac{\partial V}{\partial a_k} + D\Delta a_k. \qquad (13)$$

Here,

$$V = -\frac{1}{2}\sum_{k=1}^{M} a_k^2 + \frac{1}{4}\sum_{k=1}^{M}\sum_{\substack{k'=1 \\ k'\neq k}}^{M} a_k^2 {a'_k}^2 + \frac{1}{4}\left(\sum_{k=1}^{M} a_k^2\right)^2, \tag{14}$$

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \tag{15}$$

where, the parameter D is a diffusion coefficient and $a_k = a_k(\xi, t)$.

First, let's examine the meaning of the first term in the right hand side of eq.(13). If we neglect the second term at eq.(13), we can interpret the equation as a gradient system whose potential function V is defined by eq.(14). The potential function V makes the largest a_k in initial state converge to the value 1 and all the other a_n's ($n \neq k$) converge to the value 0 (see Fig. 5). Consequently, it leads to the state that the condition (1) is satisfied.

Secondly, we would mention the second term in the right hand side of eq.(13). As shown in the section 2.1, the diffusion term has an ability to make the value uniform (see Fig. 5). The dynamics with only the diffusion term, i.e., without the term of potential function, all the a_k's in the same element plane takes the same value as the phase, which means the condition (2) becomes to be satisfied.

By selecting the diffusion coefficient appropriately, we can make the system to satisfy both the conditions at the stable state. The diffusion coefficient D controls the trade-off between the reaction and the diffusion. Generally speaking, the above two conditions are achieved by making the diffusion act faster than the reaction. Namely, we can select the diffusion coefficient D larger.

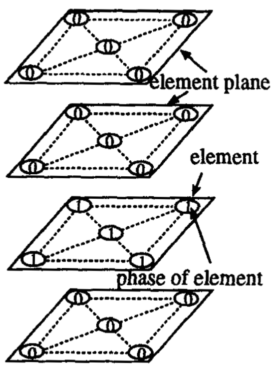


Fig. 4 Stable state

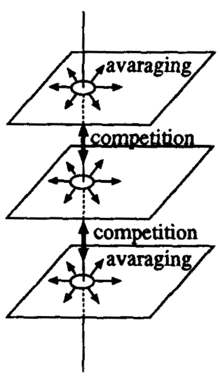


Fig. 5 Interactions

Finally, let's consider how to reconstruct the output pattern. The output pattern has to correspond one of the memorized patterns. If above two conditions are satisfied in the stable state, the correspondence between the output and the memorized picture becomes possible by the reconstruction of the output patterns as

$$p(\xi, t) = \frac{\sum_{k=1}^{M} v(k, x) a_k(\xi, t)}{\sum_{k=1}^{M} a_k(\xi, t)}, \tag{16}$$

In eq.(16), each phase plays a role of weight at the calculation of the summation in every groups.

3 Simulations

In this section we show some simulation results. The simulations based on the two different method are shown, i.e., one is the only diffusion equation and the other is by the reaction-diffusion equation.

The system memorizes five pictures in the size of 100×100 pixels with the brightness from 0 to 255 in advance (see Fig. 6).

And we set the diffusion coefficient to 200.

3.1 Recalling from the picture with high-frequency noises

First, as shown in Fig.7(a), we use the picture 1 with noise. In this example all the pixels were affected by uniformly random noise whose maximum amplitude is 40 and the average is 0.

The output converges to almost the same picture as the original picture without noise as shown in Fig.7 (b), (c). It shows that our methods have robustness for the random noises of average 0.

3.2 Recalling from a couple of mixed pictures

In order to examine the removability of the noise in the space spanned by the memorized pictures, the picture which is made from mixing the picture 2 and 5 at the ratio of 1:1 is offered to both systems (see Fig.8(a)).

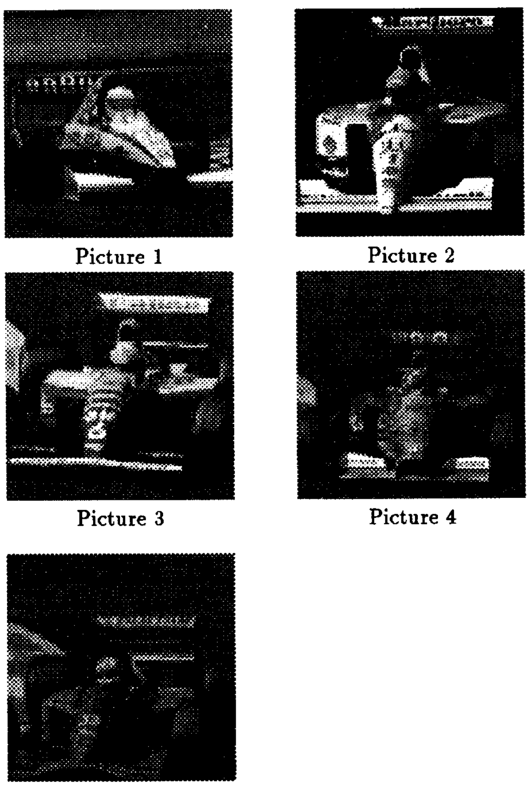
As is known from the way of construction, the output picture becomes equivalent to the input pattern in the system with diffusion equation (see Fig.8 (b)).

On the other hand, as shown in Fig.8 (c), the output by the reaction-diffusion equation converges to one of the memorized pictures. From this, it is known that the noise in the space spanned by the memorized pictures can be removed. In this example, since the ratio of mixed pictures is same, the output converge to either of both at the same probability. If the ratio is different, for example the ratio 6:4, then the stronger one will be recalled.

3.3 Recalling from a part of the picture

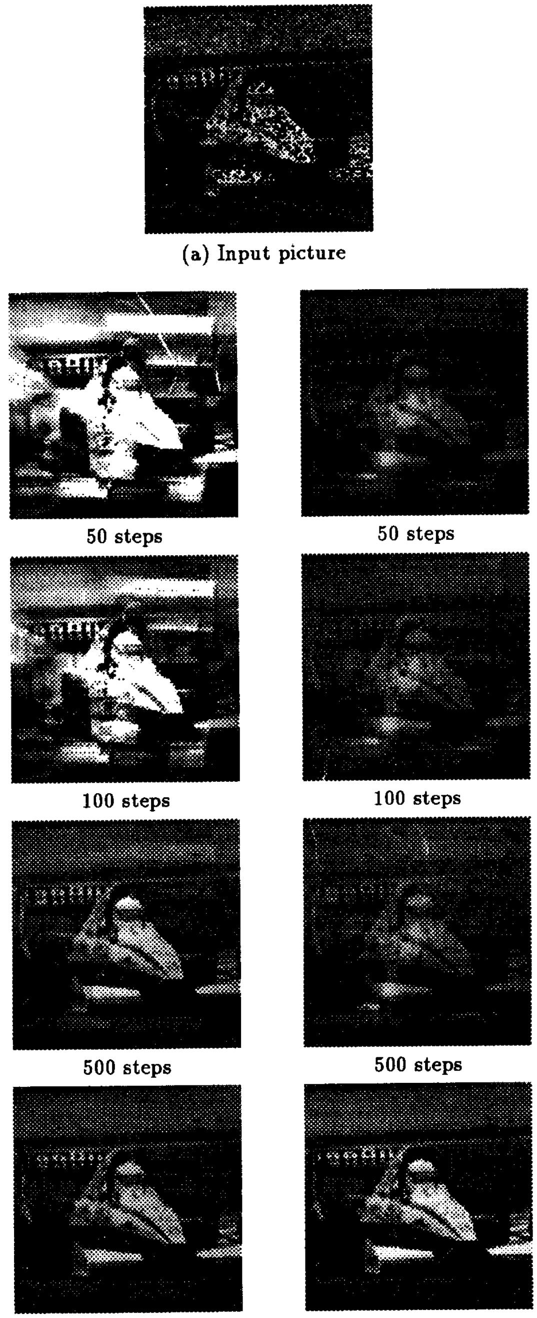
To examine the ability of association, we offer the part of the picture 2 as the input picture to both systems (see Fig.9(a)).

As shown in Fig.9, the output picture in the system with only diffusion equation includes the components other than the picture 2 (Fig.9 (b)), while the output in the reaction-diffusion equation converges to the picture 2 correctly (Fig.9 (c)).



Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

Fig. 6 Five memorized pictures.

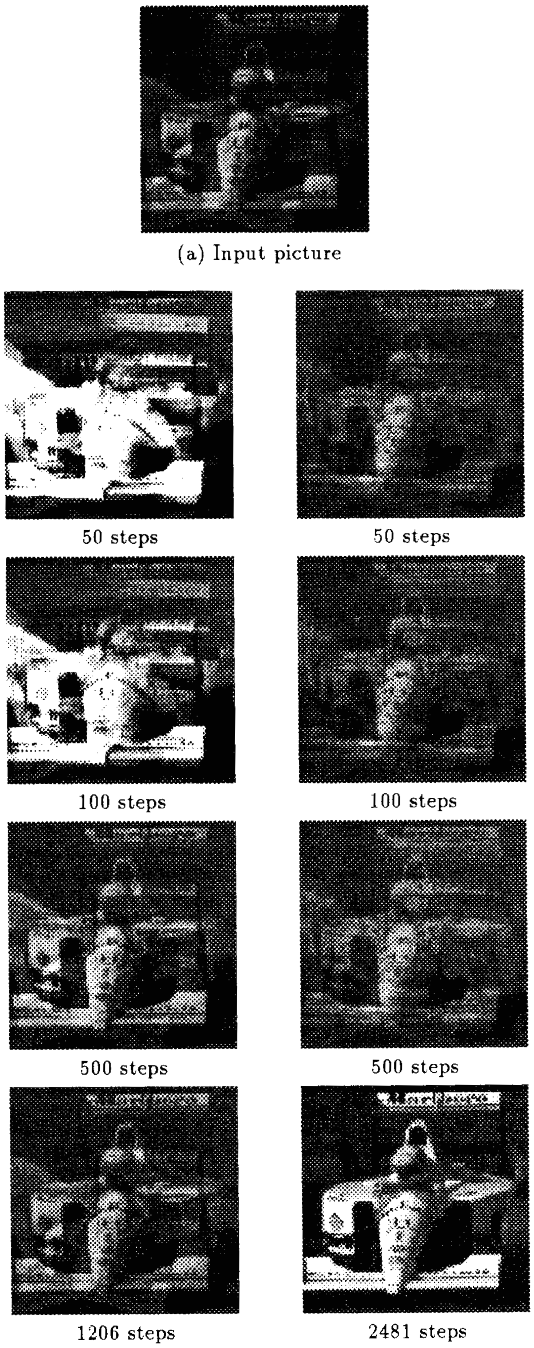


(a) Input picture

50 steps 50 steps 100 steps 100 steps 500 steps 500 steps 1143 steps 1396 steps

(b) Diffusion equation (c) Reaction-diffusion equation

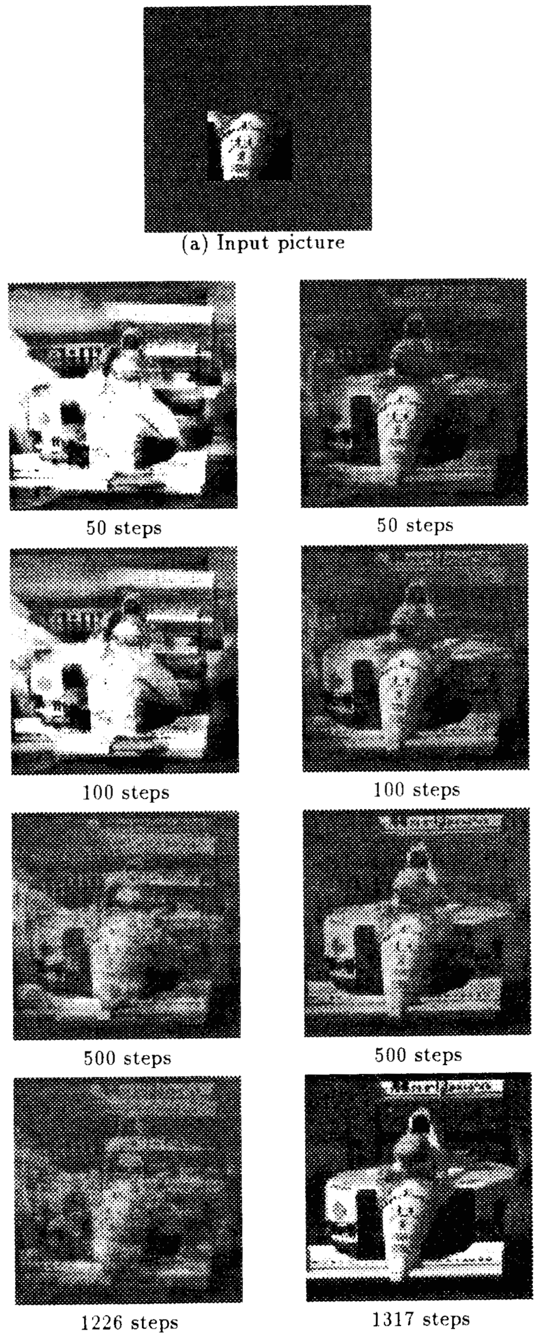
Fig. 7 Simulation results for the input with high-frequency noise



(b) Diffusion equation

(c) Reaction-diffusion equation

Fig. 8 Simulation result for the input consisted of a couple of pictures



(b) Diffusion equation

(c) Reaction-diffusion equation

Fig. 9 Simulation result for partially given input.

4 Conclusion

In this paper we proposed the pictorial pattern recognition method based on the concept of the autonomous decentralized system. As the results we can point out that the recognition system is constructed by the parallel distributed manner with less connections. Further the system with nonlinear interactions has the ability to remove the noises included in the space spanned by the memorized pictures, while the associative memory by use of the orthogonal projection cannot remove it.

Acknowledgments

We thank to Mr. Seiji Toyohara of NKK Corp. for a lot of cooperation in proceeding this research. A part of the work presented in this paper was supported by the science research fund from the Ministry of Education, Science and Culture of Japan under Grant (#04218209,06452253).

References

[1] "Special edition Autonomous decentralized systems," *Journal of the society of instrument and control engineers*, Vol. 29, No. 10, 1990 (Japanese).

[2] H.Haken, *Synergetic Computers and Cognition*, Springer-Verlag, 1990.

[3] H.Haken, *Advanced Synergetics*, Tokai University Press, 1986 (Japanese).

[4] A.Fuchs H.Haken, "Pattern Recognition and Associative Memory as a Dynamical Processes in a Synergetic System," *Biological Cybernetics*, Vol. 60, pp. 17-22, 107-109, Springer-Verlag, 1988.

[5] A.Fuchs and H.Haken, *Computer Simulations of Pattern Recognition as a Dynamical Process of a Synergetic System*, Springer-Verlag, 1988.

[6] H.Haken, *synergetic in Pattern Recognition and Associative Action*, Springer-Verlag, 1988.

[7] A.S.Mikhailov, *Foundation of Synergetics I*, Springer-Verlag, 1990.

[8] K.Nakano, *Foundation of Neuro-Computer*, Corona Publishing, 1990 (Japanese).

[9] Kohonen , "Representation of associated pairs by matrix operators," *IEEE Trans.*, C-22, pp. 701-702, 1973.

[10] K.Matsuoka, "On Various Structures of Orthogonal Projection Type of Associative Network," *IEICE Trans.*, Vol.J73-D-II, No. 4, pp. 641-647, 1990 (Japanese).