Proceedings of the 2007 IEEE/RSJ International
Conference on Intelligent Robots and Systems
San Diego, CA, USA, Oct 29 - Nov 2, 2007

Numerical Analysis for Optimal Posture of Circular Object Grasped with Frictions

Satoshi ITO*,† Yuuki Mizukoshi* and Minoru Sasaki*
* Faculty of Engineering, Gifu University, Gifu 501-1193, Japan
E-mail:satoshi@gifu-u.ac.jp
†BMC Research Center, RIKEN, Nagoya 463-0003, Japan

***Abstract*— When grasping an object, friction forces are sometimes utilized effectively. This friction forces allows us to manipulate the object to various directions. Regarding such a grasped object posture, we reported an analysis on the 2D-space grasping with two contact points. Selecting the square sum of the contact forces as an evaluation function of the object posture, we concluded that the optimal posture is the one where the midpoint of two contact points and the center of mass of the object align vertically. In this analysis, however, the friction condition is not taken into account, and thus this result is applied only to the grasp by use of fixtures, not to the one with frictions. In this paper, we aim at extending this result into the grasp with frictions. Especially, we analyze a grasp of circular object in the 2-D space in a numerical manner, and discuss the physical meaning of the optimal posture.**

Index Terms – Grasping, Friction force, Object direction, Optimization, Contact force

I. INTRODUCTION

Grasping is a fundamental action in some tasks such as conveying, assembling, and manipulating objects. The grasp in such tasks is sometimes achieved using frictions between the object and the fingers. From the aspect of the stability, the power grasp [1] or the form closure grasp[2] is superior. However, a reason why we treat the grasp with the frictions here is that it has a possibility to manipulate the object speedy and skillfully.

The grasped object can take various posture, i.e., the manipulator can change the direction of the end-effector with grasping the object. Actually, we often encounter a case where we must maintain the posture of the object to a given directions: For example, fixing a mechanical part to machinery using screws, we should keep its posture constant. But, in some other cases, especially during the transportation or conveying task, it is possible to select an arbitrary posture of the grasped object. Namely, the direction of the grasped object should be one of the design factors in the motion planning. However, the optimization of the grasped object is not discussed so much in the previous studies.

The optimization of the grasping has been reported using various method [3], [4], [5], [6], [7], [8], and many optimizing factors for grasping have been investigated such as position of contact points, internal forces, finger joint torques and so on [9]. Especially, the selection of contact points is significant for automatic grasp by a robot hand or a manipulator, and so many studies treat this problem[10], [11], [12]. However, the position of the contact points is often restricted due to task requirements or size of the hand, and thus contact points are not always be chosen freely. Taking it into the consideration, we start discussion with an assumption that a prehensible set of contact points are assigned. Finger joint torques [13] might be another important optimizing factor. However, it depend on the structure of the end-effectors. We here select the posture of the grasped object as an optimizing factor, because it is a property specific to the grasped object and independent of the structure of end-effectors.

In summary, we here consider the following problem:

- Find the optimal posture of the object grasped with frictions under the condition that contact points are assigned with which the object is grasped stably.

As an evaluation of the object posture, we adopt the norm of contact force vector. Here, the contact force vector is a vector whose element is a component of the contact forces, and the contact force is a force acting at the contact point and consists of the normal force and the friction force. To maintain the object position statically during the grasp, the contact forces must compensate the gravitational effect of the object. Then, small amount of contact forces is effective. In addition, smaller contact forces reduce the possibility for object to be broken. Thus, an evaluation is defined as the norm of contact forces. Accordingly, the problem is to find the object posture such that minimizes the square sum of the contact forces.

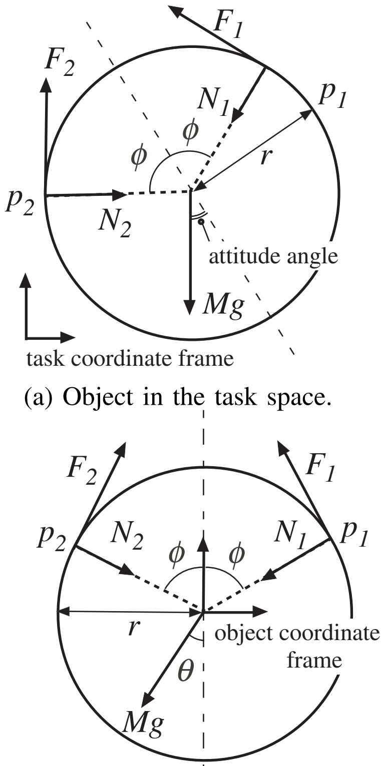
II. 2D GRASPING OF A CIRCULAR OBJECT

A. Assumptions

Throughout this paper, we set the following assumptions.

- Task space of grasping is 2 dimensional where the gravitational direction is included.
- An object is rigid and homogeneous.
- The number of the contact points is two.
- The type of the contact is a point contact with friction [14].
- At the contact points, the shape of the object is smooth.

Then, two contact points is the minimal number to grasp an object in the 2D space. Indeed, more contact points make it easier to grasp it. However, if the object can be grasped with less contact points, an extra end-effector (finger) does not be prepared, or can be utilized for the subsequent manipulation.



(a) Object in the task space.

(b) The object coordinate frame.

Fig. 1. Grasping of a circular object.

B. Formulation

This paper is the first step of the analysis for the object posture optimization in the grasping with friction. Thus, we take an circular object as the simplest example.

The circular object is homogeneous with radius r implying that the center of circle is also center of mass of this object. As shown in Fig. 1(a), two contact points are set on the object surface whose relative angle is 2ϕ ($\phi \neq 0$) at the center.

The problem is to find postural angle of object from the gravitational direction that minimize the square sum of the contact forces. The problem becomes accessible by means of introducing the object coordinate frame as shown in Fig. 1(b). In the object frame, the origin is set to the center of mass, and y axis is set in the direction to the midpoint of two contact points. Then, contact points are denoted respectively by

$$\boldsymbol{p}_1 = \begin{bmatrix} r\sin\phi & r\cos\phi \end{bmatrix}^T \tag{1}$$

$$\boldsymbol{p}_2 = \begin{bmatrix} -r\sin\phi & r\cos\phi \end{bmatrix}^T. \tag{2}$$

The object posture in the task coordinate frame is expressed as the relative direction of the gravity in the object frame, which is denoted as the clockwise angle θ from the y axis. Because of the symmetry, we can restrict the range as $0 < \phi \leq \pi/2$.

The balance of the forces and moments in the stable grasping are described by the equation,

$$L\boldsymbol{F} = \boldsymbol{M} \tag{3}$$

Here, L is a grasp matrix given as

$$L = \begin{bmatrix} -\sin\phi & -\cos\phi & \sin\phi & \cos\phi \\ -\cos\phi & \sin\phi & -\cos\phi & \sin\phi \\ 0 & r & 0 & -r \end{bmatrix} \tag{4}$$

$\boldsymbol{F}$ is a unknown contact force vector whose components are the normal force N_i and friction forceF_i (i=1,2), i.e.,

$$\boldsymbol{F} = \begin{bmatrix} N_1 & F_1 & N_2 & F_2 \end{bmatrix}^T \tag{5}$$

and $\boldsymbol{M}$ is given as

$$\boldsymbol{M} = \begin{bmatrix} Mg\sin\theta & Mg\cos\theta & 0 \end{bmatrix}^T \tag{6}$$

representing the effect of the gravity, where M is a mass of the object and g is a constant of gravitation.

The solution of the equation (3) can be written as

$$\boldsymbol{F} = \boldsymbol{F}_T(\theta) + \alpha\boldsymbol{F}_N \tag{7}$$

where

$$\boldsymbol{F}_T(\theta) = L^{\dagger}\boldsymbol{M}(\theta) \tag{8}$$

$L^{\dagger}$ is a pseudo-inverse matrix of L, $\boldsymbol{F}_N$ is a unit vector in the null space of L, i.e.,

$$L\boldsymbol{F}_N = \boldsymbol{0} \tag{9}$$

and α is a scalar corresponding to the amount of the internal forces. After some calculations, we obtain the following equations for $\boldsymbol{F}_T(\theta)$ and $\boldsymbol{F}_N$:

$$\boldsymbol{F}_T(\theta) = \frac{Mg}{2}\begin{bmatrix} -\frac{\sin\theta}{\sin\phi} - \cos\phi\cos\theta \\ \sin\phi\cos\theta \\ \frac{\sin\theta}{\sin\phi} - \cos\phi\cos\theta \\ \sin\phi\cos\theta \end{bmatrix} \tag{10}$$

$$\boldsymbol{F}_N = \frac{1}{\sqrt{2}}\begin{bmatrix} \sin\phi \\ \cos\phi \\ \sin\phi \\ \cos\phi \end{bmatrix} \tag{11}$$

Among the solutions of the equation (3), we must select the one which minimizes the following evaluation function V

$$V = \boldsymbol{F}^T\boldsymbol{F} \tag{12}$$

Substituting (12) by (7), we obtain the following equation

$$V = ||\boldsymbol{F}_T(\theta)||^2 + \alpha^2||\boldsymbol{F}_N||^2 \tag{13}$$

The problem here is to find θ that minimize (13)

C. Our previous analysis

In (13), the variable θ and α are separated to different terms. Thus, in our previous paper [16], we optimized it with respect to each variables. Regarding to α, the smaller it is, the better this evaluation function becomes. Thus, we tried to minimize (13) with respect to θ. Our mathematical analysis found that the evaluation function takes minimal value at the posture where the midpoint of the two contact forces and the center of mass of the object align on the gravitational lines. This result can be generalized to the 3D grasping of the convex object [16].

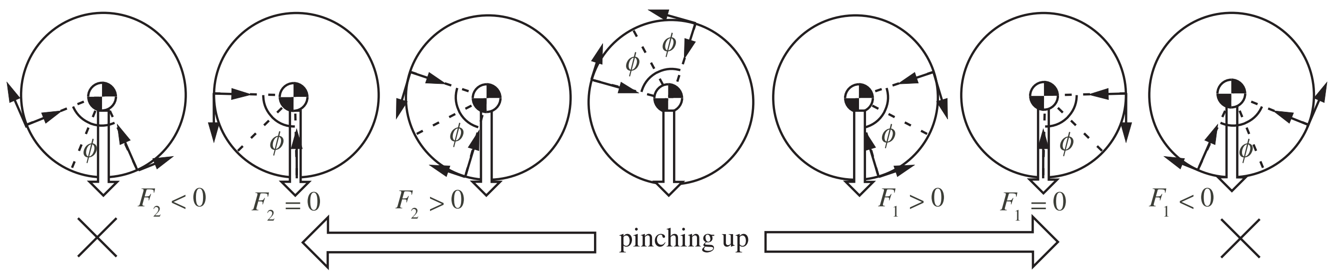


Fig. 2. Range of pinching-up grasping.

Certainly, this result is valid in the condition that contact force can be generated to any directions, e.g., for the grasp using fixture. However, it cannot be applied to the grasp with friction without modification, because the friction conditions as well as unilateral condition of normal force are not contained as the coustraints. Probable, the α would be a function of the θ due to these constraints.

From this point of view, we set the conditions for stable grasping with frictions mathematically. Next, we consider the internal force α that satisfies these conditions. Then, using this α, we obtain the optimal posture that minimize the evaluation function (13).

D. Stable Grasp Conditions

To ensure the meaningful stable grasp with frictions, three types of the conditions are required: unilateral condition of normal reaction force, pinching-up condition and friction condition.

1) unilateral condition of normal reaction force: The reaction force from the object is generally unilateral. The reaction force can push the fingers, but cannot pull them. This condition is described as follows.

$$N_1 > 0. \tag{14}$$

$$N_2 > 0. \tag{15}$$

2) pinching-up condition: When grasping an object with frictions, the friction forces may play an important role for compensating the gravity of the object. Indeed, we can consider a case in which the object is placed on the two finger tips stably. Such a case is not interesting to us, since we cannot manipulate it by such a method. To manipulate it, the object is tightly pinched by two fingers. In order to exclude the former case, the following conditions are imposed to the friction forces.

$$F_1 > 0. \tag{16}$$

$$F_2 > 0. \tag{17}$$

We call them pinching-up conditions.

3) friction condition: To grasp the object stably, the fingers should not slip on the object surface. These conditions are described as follows:

$$|\mu_1 N_1| > F_1. \tag{18}$$

$$|\mu_2 N_2| > F_2. \tag{19}$$

Here, μ_1 and μ_2 is a friction coefficient at each contact point.

E. Approach

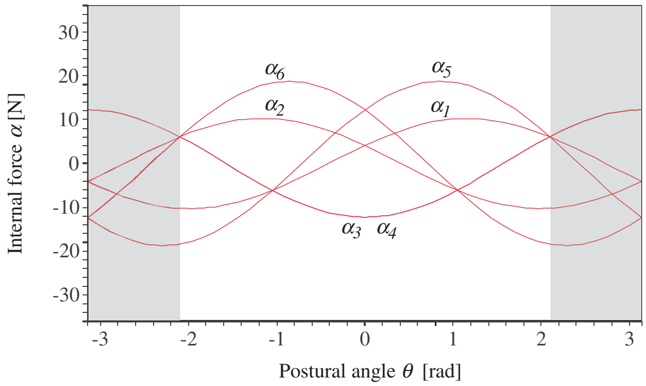
To summarize, we can describe the problem here as follows:

- Find the postural angle θ such that minimize the evaluation function (12) under the equality condition (3) as well as the inequality condition (14) - (19).

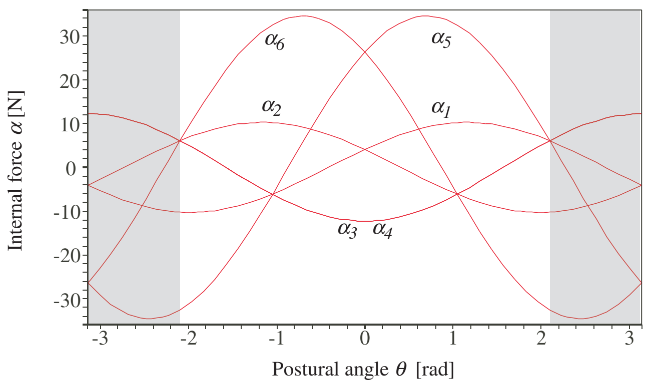
The solution will be obtained using Lagrange method. Lagrange function is constructed using Lagrange multipliers and then induces the Karusch-Kuhn-Tucker (KKT) conditions from the Lagrange function. The KKT conditions can be solved analytically by some computer mathematical tools. However, the outputted solutions are so complex and are classified by so many combinations of the case conditions that we cannot understand the significant meaning of the solutions.

Because of this reason, we take an another approach as follows. Firstly, we solve the equation (3) in the form of (7). In the section II-C, α was put to zero, considering that the smaller the internal force α provides the better evaluation. However, it is possible that the α depends on the posture θ due to the stable grasp conditions (14) - (19).

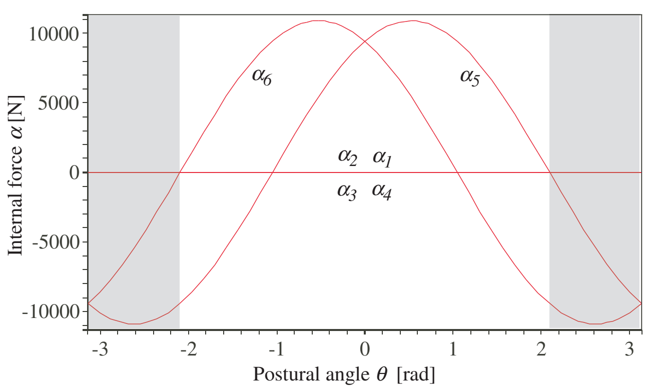
Thus, we firstly calculate the α that satisfies force balance condition (7) and one more condition: $N_1 = 0$, $N_2 = 0$, $F_1 = 0$, $F_2 = 0$, $|\mu_1 N_1| = F_1$, and $|\mu_2 N_2| = F_2$. We put such α, respectively, to α_1, α_2, α_3, α_4, α_5 and α_6, which will be expressed as a function of the θ. To satisfy the conditions (14) - (19), the feasible α in the solution of (7) must be larger than all the $\alpha_k(\theta)$ $(k = 1, \cdots, 6)$ at every θ value. Therefore, we select the maximal $\alpha_k(\theta)$ for each θ, which is denoted as $\bar{\alpha}(\theta)$ This ensures $\alpha \geq \bar{\alpha} \geq \alpha_k$ $(k = 1, \cdots, 6)$. Finally, we substitute (7) by these selected $\bar{\alpha}(\theta)$ in each corresponding range. In this way, we can obtain the solutions of (7) that also satisfy all the stable grasp conditions. Using this solution, we minimize the evaluation function (12) with respect to θ.



(a) $\mu_1 = \mu_2 = \sqrt{3}$.

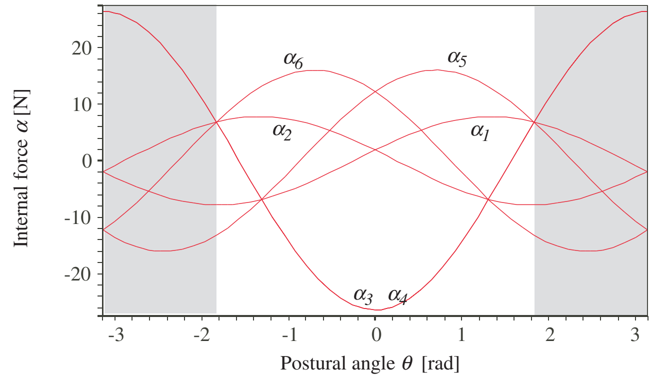


(b) $\mu_1 = \mu_2 = 1$.

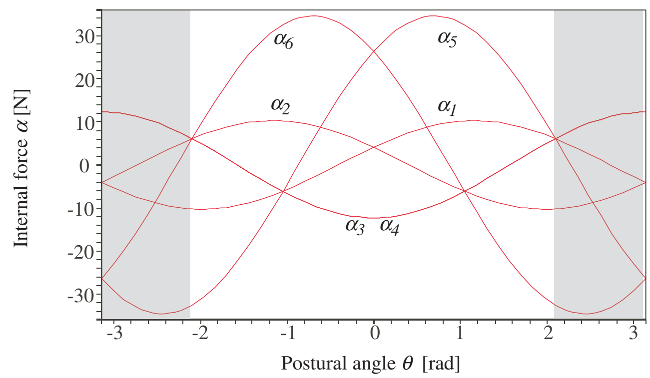


(c) $\mu_1 = \mu_2 = 1/\sqrt{3} + \epsilon$.

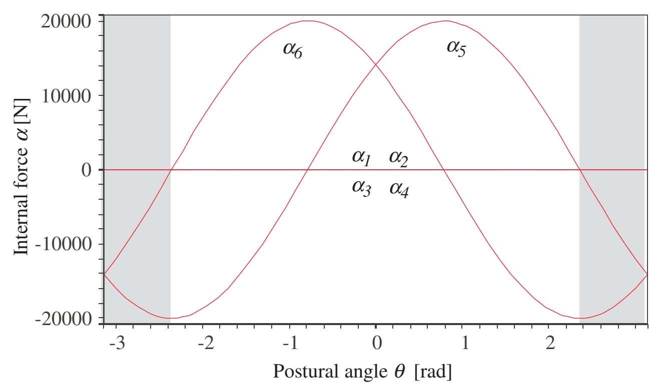
Fig. 3. Numerical analysis for $\phi = \pi/3$.



(a) $\phi = 5\pi/12$.



(b) $\phi = \pi/3$.



(c) $\phi = \pi/4$.

Fig. 4. Numerical analysis for $\mu_1 = \mu_2 = 1$.

F. Mathematical Analysis

Following the above approach, we try to solve the problem analytically. The solution that satisfies each stable grasp condition is given as follows:

$$\alpha > \frac{Mg\left(\sin\theta + \sin\phi\cos\phi\cos\theta\right)}{\sqrt{2}\sin^2\phi} \equiv \alpha_1 \tag{20}$$

$$\alpha > \frac{Mg(-\sin\theta + \sin\phi\cos\phi\cos\theta)}{\sqrt{2}\sin^2\phi} \equiv \alpha_2 \tag{21}$$

$$\alpha > \frac{Mg(\sin\phi\cos\theta)}{\sqrt{2}\cos\phi} \equiv \alpha_3 \tag{22}$$

$$\alpha > \frac{Mg(\sin\phi\cos\theta)}{\sqrt{2}\cos\phi} \equiv \alpha_4 \tag{23}$$

$$\alpha > \frac{Mg\left(\sin^2\phi\cos\theta + \mu_1\sin\theta + \mu_1\cos\phi\cos\theta\sin\phi\right)}{\sqrt{2}\sin\phi\left(\mu_1\sin\phi - \cos\phi\right)} \equiv \alpha_5 \tag{24}$$

$$\alpha > \frac{Mg\left(\sin^2\phi\cos\theta - \mu_2\sin\theta + \mu_2\cos\phi\cos\theta\sin\phi\right)}{\sqrt{2}\sin\phi\left(\mu_2\sin\phi - \cos\phi\right)} \equiv \alpha_6 \tag{25}$$

Here, we use the relation $\cos\phi > 0$, $\sin\phi > 0$, $\mu_1\sin\phi - \cos\phi > 0$ and $\mu_2\sin\phi - \cos\phi > 0$.

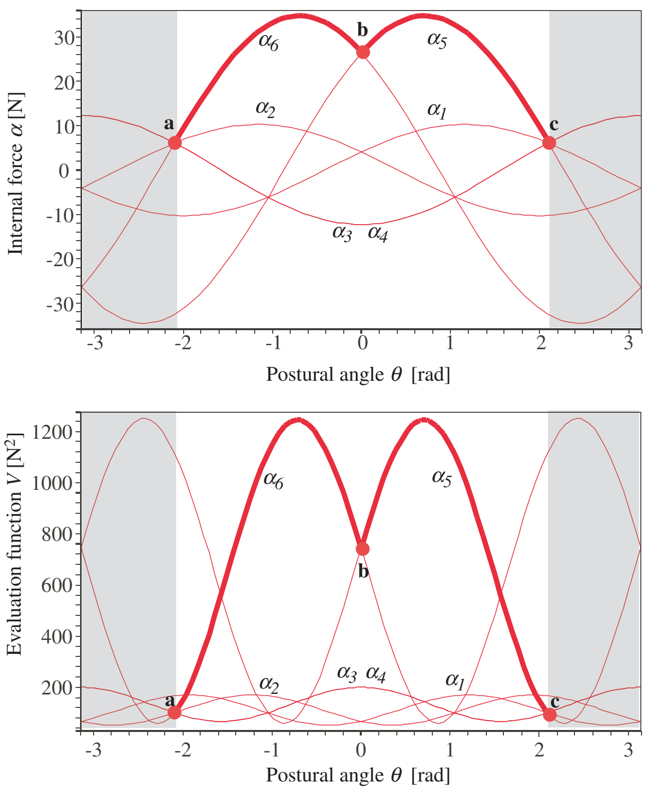


Fig. 5. Numerical analysis for $\mu_1 = \mu_2 = 1, \phi = \pi/3$.

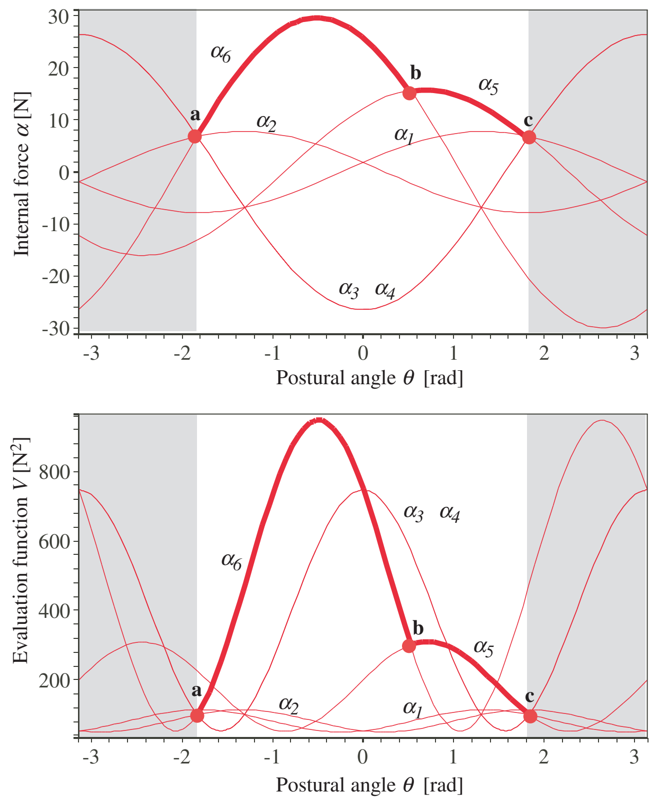


Fig. 6. Numerical analysis for $\mu_1 = 1, \mu_2 = 1/\sqrt{3}, \phi = 5\pi/12$.

Among these α_k $(k = 1, \cdots, 6)$, we have to select the maximal one at every θ. To simplify the calculation, we re-examine the conditions (14) - (19), here. In order to satisfy three inequalities $N_1 > 0$, $F_1 > 0$ and $|\mu N_1| > F_1$, it is sufficient that only two inequalities $F_1 > 0$ and $\mu_1 N_1 > F_1$ hold. It is the same for $N_2 > 0$, $F_2 > 0$ and $|\mu N_2| > F_2$. Thus, we consider the only four inequalities: $F_1 > 0$, $\mu_1 N_1 > F_1$, $F_2 > 0$ and $\mu_2 N_2 > F_2$.

In addition, we examine the pinching-up conditions. This conditions are used for excluding the situation in which the object are just placed on the finger. This purpose is achieved by restricting the range of θ. As shown is Fig. 2, restrict the range of θ in $-(\pi - \phi) < \theta < \pi - \phi$ and then the gravitational direction from the center of mass of the object does not pass between two contact points. Accordingly, the object is not placed on the fingers but grasped actively using contact forces.

In summary, to ensure a meaningful stable grasping, all the conditions we have to consider are $\mu_1 N_1 > F_1$ and $\mu_2 N_2 > F_2$ in the range $-(\pi - \phi) < \theta < \pi - \phi$.

Although the stable pinching condition is simplified well, the equations are still too complex to analyze. Thus, in the next section, we introduce the numerical method to elucidate the optimal solution.

G. Numerical Analysis

The internal force α that satisfies the force balance as well as each stable pinching condition (14) - (19) is numerically calculated. At first, fixing the position of the contact points, the effect of the friction coefficient is examined. The center angle of two contact points 2ϕ is set to $\pi/3$, and the three different values are set to the friction coefficient μ_1 and μ_2, i.e., $\sqrt{3}$, 1 and $1/\sqrt{3} + \epsilon$. The results are shown in Fig. 3 where Mg is set to 10. The internal force α in the upper area of all the curve α_k $(k = 1, \cdots, 6)$ ensures the stable grasp. According to the section II-F, it is sufficient to consider α_5 and α_6 corresponding to the condition $\mu_1 N_1 > F_1$ and $\mu_2 N_2 > F_2$ respectively in the unshaded range $-(\pi - \phi) < \theta < \pi - \phi$. This fact is confirmed graphically from the Fig. 3.

Next, fixing the friction coefficient $\mu_1 = \mu_2 = 1$, the effect of contact point position are examined. The three different values are set to the parameter for contact position 2ϕ, i.e., $5\pi/12$, $\pi/3$ and $\pi/4$. The results are shown in Fig. 4, where $Mg = 10$. The graphs similar to Fig. 3 are obtained.

Although the magnitude of the each graphs are different in every conditions, the fact that two minimal points at $\theta = -(\pi - \phi)$ and $\theta = \pi - \phi$, as well as one local minimum point at $\theta = 0$ is commonly observed.

Secondly, the evaluation function (12) is computed for each α_k $(k = 1, \cdots, 6)$. At the first example, the parameters are set as $\mu_1 = \mu_2 = 1$, $\phi = \pi/3$ and $Mg = 10$. The upper graph shown in Fig. 5 denotes the internal force α, which is the same as the middle graph in Fig. 3 and Fig. 4. On the other hand, the evaluation function is depicted in the bottom. Among six curves, the only two graphs are significant in the range $-(\pi - \phi) < \theta < \pi - \phi$, i.e., the graphs calculated from

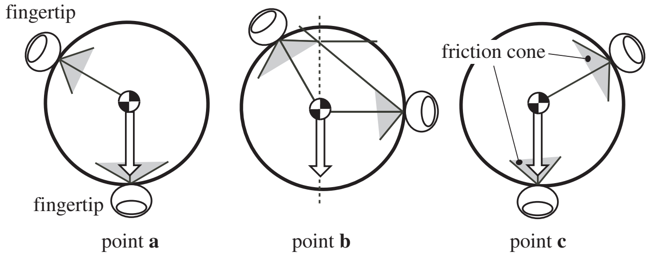


Fig. 7. The postures at the minimal points.

α_5 and α_6. As the same as the internal force, there are two minimal points **a** and **c** at the boundary and one local minimal point **b** at $\theta = 0$.

The next example is a case in which the friction coefficients are different at two contact points. The results are shown in Fig. 6. The upper graph denotes the internal force α, while the bottom does for the evaluation function. The parameters are set as $\mu_1 = 1$, $\mu_2 = 1/\sqrt{3}$, $\phi = 5\pi/12$ and $Mg = 10$. Compared to the condition in Fig. 4(a), only the μ_2 is different. Therefore, the graph of α_6 solely varies. However, the value of α_6 is crucial for determining the minimum of the evaluation function, because not only the internal force α but also the evaluation function becomes asymmetrical as shown in Fig. 6(b). In Fig. 6, two minimal points **a** and **c** at the boundaries and the local minimal point **b** between them are also observed. It implies that the features of grasp with frictions do not drastically change with small parameter deviation.

H. Physical meaning of minimal posture

Finally, the posture of the minimal point is investigated. It is trivial that, at the minimal point **a** and **c**, one of the contact points is positioned directly underneath of the center of mass of the object, as shown in Fig. 7. However, they are not the exact grasping we expected, since the object is just placed on the one contact point. The solution we are interested in is the one at the minimal point **b**. After some calculations, this is a posture such as illustrated middle in Fig. 7. In this posture, the intersecting point of the surface of two friction cone and the center of mass of the object are aligned on the gravitational direction.

III. CONCLUSION

In this paper, the optimal posture of the circular object grasped with frictions is considered. As an evaluation, the square sum of the contact forces is adopted. In addition to the force balance condition, three other conditions are imposed to ensure the meaningful stable grasp with frictions, i.e., the unilateral condition of normal force, pinching-up condition and friction condition. For numerical analysis, the internal force necessary to satisfy the stable grasp conditions are computed at first, and then the evaluation function is calculated based on these computed internal force values. Comparing these evaluation functions, one meaningful optimal solution is obtained. Some analytical calculations reveal that this posture is the one in which the intersecting point of the surface of two friction cone and the center of mass of the object are aligned on the gravitational direction. Although this result is derived from the grasping of the circular object, we expect that it can be extended to the general 2D grasping of the non-circular object. This extension will be presented in the next paper.

REFERENCES

[1] X. Y. Zhang, Y. Nakamura, K. Goda, K. Yoshimoto: Robustness of Power Grasp. Proc. of ICRA1994: 2828-2835, 1994
[2] Antonio Bicchi: On the Closure Properties of Robotic Grasping, I. J of Robotics Research, 14 (4), 319-334, 1995
[3] Fan-Tien Cheng and David E. Orin: Efficient Algorithm for Optimal Force Distribution – The Compact-Dual LP Method, IEEE Trans. on Robotics and Automation, Vol.6, No. 2, pp.178-187, (1990)
[4] Yun-Hui Liu: Qualitative Test and Force Optimization of 3-D Frictional Form-Closure Grasps Using Linear Programming, IEEE Trans. on Robotics and Automation, Vol.15, No. 1, pp.163-173, (1999)
[5] Dan Ding, Yun-Hui Liu, Michael Yu Wang and Shogun Wang: Automatic Selection of Fixturing Surfaces and Fixturing Points for Polyhedral Workpieces, IEEE Trans. on Robotics and Automation, Vol.17, No. 6, pp.833-841, (2001)
[6] Youshen Xia, Jun Wang and Lo-Ming Fok: Grasping-Force Optimization for Multifingered Robotic Hands Using a Recurrent Neural Network, IEEE Trans. on Robotics and Automation, Vol. 20, No. 3, Jun 2004
[7] E.A. Al-Gallaf: Multi-fingered robot hand optimal task force distribution neural inverse kinematics approach, Robotics and Autonomous Systems, Vol. 54 pp.34-51 (2006)
[8] Venketesh N. Dubey, Richard M. Crowder and Paul H. Chappell: Optimal object grasp using tactile sensors and fuzzy logic, Robotica, Vol. 17, pp. 685-693 (1999)
[9] K. B. Shimoga: Robot Grasp Synthesis Algorithms: A Survey, I. J of Robotics Research, 15 (3), 230-266, 1996
[10] V. D. Nguyen: Constructing Force Closure Grasp, I. J of Robotics Research, 7 (3), 3–16, 1989
[11] X. Markenscoff and C. H. Papadimitriou: Optimal Grip of a Polygon, I. J of Robotics Research, 8 (2), 17–29, 1989
[12] L. Mangialardi, G. Mantriota and A. Trentadue: A Three-dimensional criterion for the determination of optimal grip points, Robotics and Computer-integrated manufacturing, Vol. 12, No. 2, pp. 157-167 (1996)
[13] T. Watanabe and T. Yoshikawa: Optimization of Grasping by Using a Required External Force Set, Proc. of ICRA2003, 1127–1132, 2003.
[14] T.Yoshikawa et al.: Foundations of Grasping and Manipulation (in Japanese), J. of Robotic Society of Japan,Vol.13-Vol.14, 1995
[15] H. Kodama and N. Suda: Matrix theorem for system control (in Japanese), Society of Instrument and Control Engineers. 1978
[16] Satoshi ITO, Yuuki Mizukoshi, Koji Ishihara Minoru Sasaki, Optimal Direction of Grasped Object Minimizing Contact Forces, Proc. of IROS2006, pp. 1588-1593, 2006 [