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A Standing Posture Control Based on Ground Reaction Force

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Abstract

Several forces are always exerted to the locomotion systems such as inertial forces, gravitational forces, and ground reaction forces. Among them, we here focus on the ground reaction forces to consider the balance control of the simple two-link model in the sagittal plane, which contains one actuator at the ankle and two force sensors on the sole. To keep the balance, it is necessary that the ground reaction forces are kept positive at both ends of the foot, i.e., heel and toe. To achieve it, we design two control laws for each ground reaction force. By alternatively switching them, the standing posture can be maintained. Examining the behavior in the phase plane, the stability of the control laws are considered. Furthermore, we also study the posture changes by the stationery external forces in the horizontal direction. Introducing the torque input changing with large time scale, we realize the postural adjustment whereby the body is adaptively inclined to the opposite direction of the external force. By computer simulations, we confirm the convergence of such a posture adjustment.

1 Introduction

Legged systems take an advantage in the point that they can walk on the irregular ground where wheeled systems cannot move. However, such a fascinating ability is not realized without the advanced balance control. Thus, the balance control has been discussed as a main problem of locomotion systems. Especially, biped locomotion models are widely examined, because there is a small number of the legs to use for the balance keeping and so the possibility of tumbling is higher than any other multi-legged systems.

Broadly speaking, there are two powerful methods for balance control of biped system. One pays an attention to supporting leg exchanges[1]. The biped system may tumble in a moment, but, by exchanging the supporting legs before completely falling flat to the ground, the locomotion is dynamically maintained. This method is mostly applied to the one with free ankle joint. The other follows the zero moment point (ZMP) criterion [2]. The ZMP is a point on the level ground, where the total torque generated by both inertial and gravitational forces becomes zero. If the ZMP exists under the foot, the locomotion system does not tumble. Thus, the desired motions are planned so that the ZMP criterion is satisfied, and then the controller is designed to realize such desired motions.

As found in the ZMP definition, the balance naturally depends on the relation among forces exerting to the locomotion system and thus many works pay attentions to them: some compensate the actual ZMP according to the force measurement (e.g. [3]) or some design the impedance around the desired leg's position (e.g. [4]). However, these are fundamentally based on the positional information (i.e. ZMP trajectory or impedance center) and so do not directory control forces. From such a point of view, we focus the force control to keep the balance, i.e., regard ground reaction forces as controlled variables. To begin with, we limit the problem of the standing posture control. It is an important issue since the standing is the first step of locomotion.

2 Control of ground reaction forces

2.1 Simple link model

The stability of the locomotion system is determined by the positional relation between ZMP and a support pattern [5]. The support pattern is a convex polygon with minimum area which contains all the contacting points of legs on the ground. If the support pattern includes the ZMP, the balance can be kept at this instance. According to this discrimination, the single support phase of biped locomotion is most difficult to maintain the balance, since the support pattern is the smallest.

In order to treat such a difficult case as simply as possible, we suppose a model in the sagittal plane on the level ground, as shown in Fig. 1. This model consists of two links, the foot part and the body part. These two links are connected at the ankle joint with an actuator. The total weight is supported at the two point of the foot in both ends, i.e., the toe and heel. Two force sensors are attached there, which detect the vertical ground reaction forces F_T (at the toe) and F_H (at the heel).

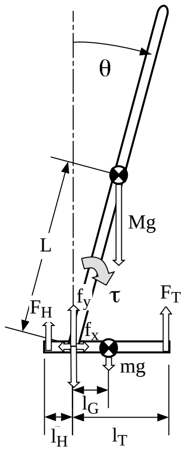


Figure 1: Link model.

The model tumbles, if either F_T or F_H becomes zero. Therefore, the object of control is to keep both F_T and F_H positive using ankle actuator. Assuming that the friction on the ground is so large that the foot does not slip on it, only the body part has dynamics:

$$I\ddot{\theta} = MLg\sin\theta + \tau, \tag{1}$$

where M is a mass of the body part, I is an inertia moment around the ankle joint, g is a gravitational acceleration, θ is an angle from the vertical direction, L is the length between ankle joint and the COG of the body part and τ is an ankle joint torque. In addition, the constraint force exerted at the ankle joint f_x and f_y is described as

$$f_x = ML\ddot{\theta}\cos\theta - ML\dot{\theta}^2\sin\theta \tag{2}$$

$$f_y = -ML\ddot{\theta}\sin\theta - ML\dot{\theta}^2\cos\theta + Mg \tag{3}$$

On the other hand, the ground reaction forces F_T and F_H become

$$F_T = -\frac{1}{\ell_T + \ell_H}\tau + m_T g + \frac{\ell_H}{\ell_T + \ell_H} f_y \tag{4}$$

$$F_H = \frac{1}{\ell_T + \ell_H}\tau + m_H g + \frac{\ell_L}{\ell_T + \ell_H} f_y \tag{5}$$

which are derived from static torqu balance around heel and toe. Here, ℓ_T, ℓ_H and ℓ_G represent the length from the ankle joint to, respectively, toe, heel and COG of foot part. m_T and m_H is a mass of foot part weighted respectively to toe and heel, which is given by

$$m_T = \frac{\ell_H + \ell_G}{\ell_T + \ell_H} m, m_H = \frac{\ell_T - \ell_G}{\ell_T + \ell_H} m \tag{6}$$

where m is the total mass of the foot part.

2.2 Two control laws

Being about to tumble, human puts their weight on the leaning side of the foot. As a result of this action, the ground reaction force increase at this side. Thus, we first examine how large they can be in order to consider the control law based on ground reaction forces. To begin with, let us study the case of F_T. Statically, if the total weight of the body part is put to the toe, then F_T takes the largest value:

$$F_T^{max} = (M + m_T)g \tag{7}$$

Accordingly, if the body part is leaning to the toe side, we control the ground reaction force F_T to converge to F_T^{max}. Such a control input is given by

$$\begin{aligned} \tau &= -(\ell_T + \ell_H)[F_T^{max} + K\int(F_T^{max} - F_T)dt] \\ &\quad +(\ell_T + \ell_H)m_T g + f_y \ell_H, \end{aligned} \tag{8}$$

since the dynamics of F_T is described as

$$F_T = F_T^{max} + K\int(F_T^{max} - F_T)dt. \tag{9}$$

In (8) or (9, $K > 0$ is a force feedback gain.

Next, let us consider the system behavior when the torque (8) is inputted. To simplify the calculation, we assume that the F_T have already controlled to F_T^{max}. Furthermore, we approximate f_y by Mg, which means that the effect of centrifugal and inertial force is small. Then, τ becomes

$$\begin{aligned} \tau &= -(\ell_T + \ell_H)F_T^{max} + (\ell_T + \ell_H)m_T g + \ell_H Mg \\ &= -M\ell_T g (= \tau_T) \end{aligned} \tag{10}$$

This torque input is advantageous since it is calculated without feedback information. Substituting it into (1), the dynamics of body part becomes

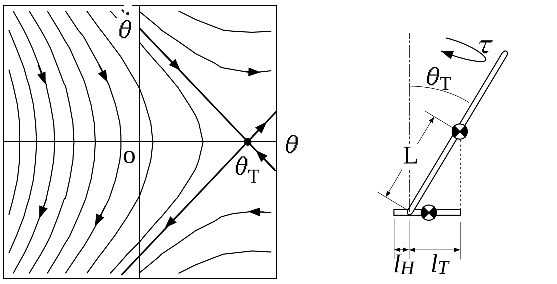
$$I\ddot{\theta} = MLg\sin\theta - M\ell_T g. \tag{11}$$

The fixed point of this dynamics in the $\theta - \dot{\theta}$ phase plane is given as $(\theta_T, 0)$ in the range $\theta \in [-\pi/2, \pi/2]$, where θ_T is a value in $[-\pi/2, \pi/2]$ that satisfies

$$\sin\theta_T = \frac{\ell_T}{L}. \tag{12}$$

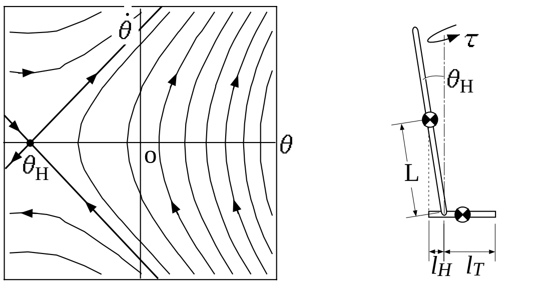
We can show this fixed point is a saddle point from the eigenvalues of linearized dynamics. The orbits of this dynamics in the phase plane are shown in Fig. 2(a).

When $\theta = \theta_T$, the COG of the body part is located just above the toe. This corresponds to the foremost



(a) $\theta - \dot{\theta}$ phase plane (b) foremost position

Figure 2: Control by τ_T



(a) $\theta - \dot{\theta}$ phase plane (b) rearmost position

Figure 3: Control by τ_H

position at which the model can statically keep balance. Therefore, controlling F_T to F_T^{max} is equivalent to making foremost position an unstable saddle point and raising up the body part from the leaning position, as shown in Fig. 2 (b).

In the case of F_H, we can derive the same argument. The desired force value is set to

$$F_H^{max} = (M + m_H)g \tag{13}$$

and then the torque input is determined by

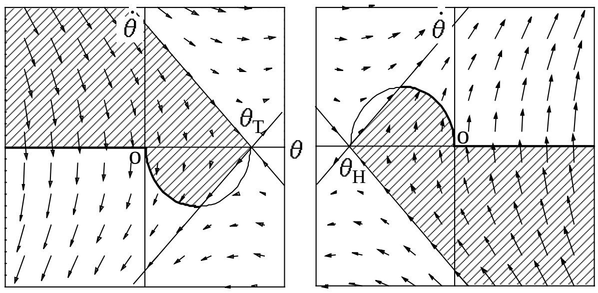
$$\begin{aligned}\tau &= (\ell_T + \ell_H)[F_H^{max} + K\int(F_H^{max} - F_H)dt] \\ &\quad -(\ell_T + \ell_H)m_H g - \ell_T f_y. \end{aligned} \tag{14}$$

The simplified torque input, which corresponds to (10), can be defined as

$$\begin{aligned}\tau &= (\ell_T + \ell_H)F_H^{max} - (\ell_T + \ell_H)m_H g - \ell_T M g \\ &= M\ell_H g (= \tau_H)\end{aligned} \tag{15}$$

The obits in the $\theta - \dot{\theta}$ phase plane is shown in Fig. 3 (a). This torque input makes $\theta = \theta_H$ unstable fixed point (saddle point) in the $\theta - \dot{\theta}$ phase plane, where θ_H is a value in $[-\pi/2, \pi/2]$ which satisfies

$$\sin\theta_H = -\frac{\ell_H}{L}. \tag{16}$$



(a) $\tau = \tau_T$ (b) $\tau = \tau_H$

Figure 4: Vector Field and switching condition.

The position at $\theta = \theta_H$ corresponds to the rearmost position at which the model can keep static balance, as depicted in Fig. 3 (b).

2.3 Switching rules

In order to make the body part upright, we switch the control laws alternatively between (10) and (15). Here, we discuss the switching rules between them. As mentioned in the previous section, τ_T acts so that the body part goes back to the heel side. Thus, when the body part is moving to the toe side, i.e., $\dot{\theta} > 0$, we adopt the control law (10) in order to stop it tumbling to the toe side. However, if it starts moving to the heel side, i.e., $\dot{\theta} < 0$, then the control law should be switched from τ_T to τ_H before long.

When it is switched is determined based on the ground reaction force. As far as τ_T is inputted, F_T will be controlled to F_T^{max} according to the derivation process of it. Therefore, we pay attention only to the positiveness of F_H. If F_T is controlled to F_T^{max} by (10), F_H will be equal to $m_H g$. So, we switch the control laws the instance that F_H becomes smaller than $m_H g$. In other words, as long as the condition

$$F_H > m_H g \tag{17}$$

is satisfied in the region $\dot{\theta} < 0$, the control input is τ_T. Using (1), (3), (5), (6) and (10), this condition changes to

$$\dot{\theta} > -\sqrt{-\frac{MgL}{I}(\sin\theta - \frac{\ell_T}{L})\tan\theta}. \tag{18}$$

In summary, the input (10) is valid in the shaded region in Fig. 4(a): when the system state transverses the bold line, the control input is switched from (10) to (15). It should be emphasized that this switching condition is defined with the force information, i.e., (17), not (18).

Similarly, we define the switching rule from (15) to (10). The equations corresponding to (17) and (18) are respectively given as

$$F_T > m_T g \tag{19}$$

and

$$\dot{\theta} < \sqrt{-\frac{MgL}{I}(\sin\theta + \frac{\ell_H}{L})\tan\theta}. \tag{20}$$

The shaded region in Fig. 4(b) is the one where τ_H is used as the control input. When the system state transverses the bold line, the control input is switched from (15) to (10)

By switching two vector field, i.e., Fig. 4(a) and 4(b), an attractor, is formed at the neighborhood of the origin. The convergence of this control law is discussed in Appendix.

3 Posture adjustment to external force

The control law proposed in previous section always makes the body part upright. However, human changes its posture with the environmental conditions, and so the body part is not always upright at the steady state. For example, when the external force is stationary added, e.g., by the strong wind, human incline the body to the windward side.

Let us consider such a behavior. Putting an horizontal external force to F_{ext}, the motion equation changes to

$$I\ddot{\theta} = MgL\sin\theta + F_{ext}L\cos\theta + \tau. \tag{21}$$

Here, we focus on the integral of ground reaction forces during a longer period T_c than the switching interval of control laws. Denoting them by $\bar{F}_H$ and $\bar{F}_T$, then

$$\bar{F}_T = \frac{1}{T_c}\int_{T_c} F_T dt = -\frac{\bar{\tau}}{\ell_T + \ell_H} + m_T g + \frac{\ell_H Mg}{\ell_T + \ell_H} \tag{22}$$

$$\bar{F}_H = \frac{1}{T_c}\int_{T_c} F_T dt = \frac{\bar{\tau}}{\ell_T + \ell_H} + m_H g + \frac{\ell_L Mg}{\ell_T + \ell_H} \tag{23}$$

where $\bar{\tau} = \frac{1}{T_c}\int_{T_c}\tau dt$, and $f_y = Mg$ assuming that the dynamics have already converged to the steady state. To keep upright against the horizontal force from the toe side (then $F_{ext} < 0$), $\bar{F}_H$ will be larger than $\bar{F}_T$. This situation implies that the weight does not put evenly to the foot, and so it is easy to tumble to the heel side than to the toe side. Accordingly, we consider the torque input τ_α to reduce the difference between $\bar{F}_H$ and $\bar{F}_T$,

$$\bar{F}_H - \bar{F}_T = \frac{2}{\ell_T + \ell_H}(\bar{\tau} + \tau_\alpha) + F_o \tag{24}$$

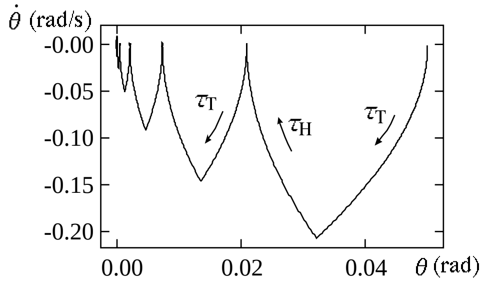


Figure 5: θ-$\dot{\theta}$ plane whose initial state is (0.05, 0).

where F_o is a constant difference resulting from the leg structure,

$$F_o = (m_H - m_T)gT_c + \frac{\ell_T - \ell_H}{\ell_T + \ell_H}MgT_c. \tag{25}$$

If τ_α is defined discreetly at every T_c as

$$\begin{aligned}\tau_\alpha(k) &= \tau_\alpha(k-1)\\ &+ K_\alpha\frac{\ell_T + \ell_L}{2}\left(F_o - \frac{\bar{E}_f(k) + \bar{E}_f(k-1)}{2}\right)T_c\end{aligned} \tag{26}$$

then $\bar{F}_H - \bar{F}_T$ converge to F_o. Here, $\bar{E}_f(k) = \bar{F}_H(k) - \bar{F}_T(k)$, k is the number of the steps of the period T_c, and K_α is a feedback gain. We use bilinear approximation to calculate the integral.

Note that τ_α adjust the offset of the switching torque τ_T and τ_H. That is, τ_T is replaces by $\tau_\alpha - Mg\ell_T$, while τ_H does by $\tau_\alpha + Mg\ell_H$.

4 Simulations

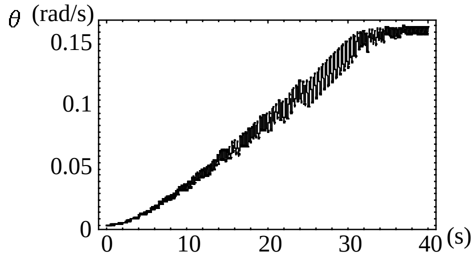
4.1 Stability

In order to examine the stability of the control laws, we set the initial state as a posture inclined to the toe side $(\theta, \dot{\theta}) = (0.05, 0)$. The parameters are set as $M = 50$, $m = 1.0$, $L = 0.85$, $\ell_T = \ell_H = 0.13$, $\ell_G = 0$. We use (10) and (15) as the control inputs. The step size of simulation is set to 0.001(s), and the simulating time is 1.0(s). As shown in Fig. 5, the system state converges to the attractor at the neighborhood of the origin in θ-$\dot{\theta}$ plane, i.e., upright posture.

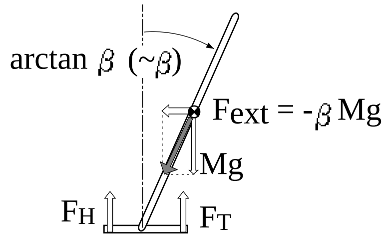
4.2 Posture changes

Next, we give a horizontal external force as follows:

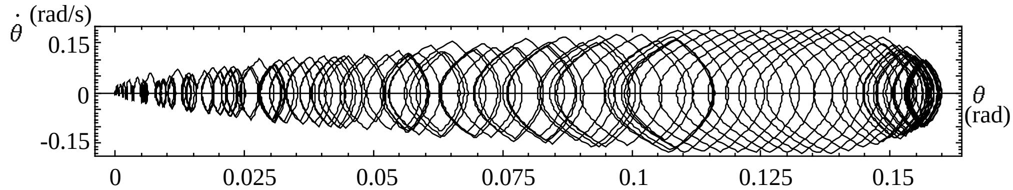
$$f_{ext} = \begin{cases} -\beta Mgt/30 & (0 \leq t < 30) \\ -\beta Mg & (t \geq 30) \end{cases} \tag{27}$$



(a) Time evolution of θ with τ_α.



(b) Stationary posture



(c) Behavior in the θ-$\dot{\theta}$ plane with τ_α.

Figure 6: The effect of external force.

Here, β is set to 0.16 and t denotes the time. This external force tumbles the model on the floor to the heel side, if using a conventional position feedback law which is designed to control the body's angle θ to vertical position 0 (rad). This is because the ZMP calculated from the gravitational and external forces goes out of foot, i.e., slightly backward to the heel. Although it is not shown, the body actually tumbles down to the heel side with the control laws used in the simulation of section 4.1.

However, introducing an adaptive torque τ_α, the body part is gradually inclined to the toe side, as a result of which the ZMP is kept under the foot. Fig. 6(a) shows the time evolution of the angle θ, which changes with the magnitude of the external force. In this simulation, the initial state is set to $(\theta, \dot{\theta}) = (0, 0)$, $K_\alpha = 0.01$, $T_c = 0.5$ and the simulating time is $40.0(s)$. The other parameters are the same as the simulation in the previous section.

At the end, θ converges about to $0.16(\sim \arctan 0.16)$. Fig. 6(b) shows the posture in the stationary state, where the gravitational force and the external force are balanced and thus generate no torque around the ankle. Therefore, it is a preferable posture from the energetic point of view. The stability of this posture is also maintained by switching two control laws, $\tau_T + \tau_\alpha$ and $\tau_H + \tau_\alpha$. Thus, the vibration is observed in Fig. 6(a). Fig. 6(c) shows the behavior in the θ-$\dot{\theta}$ phase plane. Needless to say, the ground reaction forces, F_T and F_H are kept positive during the simulation.

5 Conclusion

In this paper, we considered a balance control of a simple legged system based on the ground reaction force. First, we define two force control laws: one controls the ground reaction force exerted at the heel, the other does at the toe. Then, by alternatively switching them, an attractor is formed near the upright position. Furthermore, we introduced a control input varying in a large time scale. This adaptive torque input acts so that the difference of the time integral in two ground reaction forces converges to the value without any external forces. As a result, the standing posture is gradually inclined against the external force.

In our control laws, the joint angle is not a controlled variable. Thus, any desired postures are not given in advance. Instead, the environmental conditions, i.e., the external forces, determine the stationary standing posture such that the time average of the ankle torque become smaller.

As a future works, we should show the convergence of this postural adjustment. A part of this research was financed by the Research Foundation for the Electrotechnology of Chubu (R-11102).

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Appendix

We discuss the convergence of control laws defined in section 2. Here, we treat the simple case $\ell_H = \ell_T = \ell$, which means that the ankle joint is located at the center of the foot part. Then, the following two lemma holds.

Lemma 1 *For the dynamical system defined by (1) and (10), the initial condition is set to* $(\theta_0, 0)$*, where* $(0 < \theta_0 < \theta_T)$*. Then, the orbit of solution calculated from Hamiltonian*

$$\frac{MgL}{I}(\cos\theta + \frac{\ell}{L}\theta) + \frac{1}{2}\dot{\theta}^2 = \frac{MgL}{I}(\cos\theta_0 + \frac{\ell}{L}\theta_0) \quad (28)$$

intersects the curve specified by $F_H = m_H g$*, i.e.,*

$$\dot{\theta} = -\sqrt{-\frac{MgL}{I}(\sin\theta - \frac{\ell}{L})\tan\theta}. \quad (29)$$

in the $\theta - \dot{\theta}$ *plane.*

Lemma 2 *Consider the dynamical systems defined by (1) and (15). At all the points on (29), the vector field points to the inwards (upper ward) direction of (29).*

Now let us consider the system behavior. Assume that the initial state is $(\theta_0, 0)$ where $0 < \theta_0 < \theta_T$. According to the lemma 1, the orbit of the solution (28) intersects (29). Put this point to $(\tilde{\theta}_0, \dot{\tilde{\theta}}_0)$. Here, $\tilde{\theta}_0 < \theta_0$ holds due to the monotonousness of the orbit (28). When the state reaches the point $(\tilde{\theta}_0, \dot{\tilde{\theta}}_0)$, the control law (10) is immediately switched to (15). According to Lemma 2, the time evolution followed by (15) is directed upward of (29). Thus, the orbit of the solution obtained from Hamiltonian

$$\frac{MgL}{I}(\cos\theta - \frac{\ell}{L}) + \frac{1}{2}\dot{\theta}^2 = \frac{MgL}{I}(\cos\tilde{\theta}_0 - \frac{\ell}{L}) + \frac{1}{2}\dot{\tilde{\theta}}_0^{\,2} \quad (30)$$

encounters the θ axis without intersecting (29) again. Putting this cross point to $(\theta_1, 0)$, $\theta_1 < \tilde{\theta}_0$ is satisfied from the monotonousness of the orbit given by (30). At this point $(\theta_1, 0)$, the control law (15) is again switched back to (10). Here, regarding the above process as the one cycle, construct the return map whose transection is θ axis. From the above discussion, $0 < \theta_1 < \tilde{\theta}_0 < \theta_0$ is satisfied, and so $\theta_1 = \gamma_0\theta_0$ $(0 < \gamma_0 < 1)$. Generally, the return map is described as

$$\theta_{n+1} = \gamma_n\theta_n (0 < \gamma_n < 1). \quad (31)$$

Therefore, θ_n is represented by

$$\theta_n = \gamma_{n-1}\theta_{n-1} = \cdots = \prod_{i=0}^{n-1}\gamma_i\theta_0 < \gamma^n\theta_0 \quad (32)$$

where $\gamma = \max\gamma_i$, Now, we can easily show that θ_n goes to 0 with $n \to \infty$, since $0 < \gamma < 1$. This means that the body part converges to the upright position by the control lows in section 2.

From the symmetry of the system, we can deduce the same result for the initial state $(\theta_0, 0)(\theta_H < \theta_0 < 0)$. These results are summarized to the following theorem:

Theorem 1 *For the dynamical system described as (1), the control laws are defined as (10) and (15) whose switching conditions are illustrated as a bold line on the* θ*-*$\dot{\theta}$ *phase plane in Fig. 4. If* $\ell_T = \ell_H$*, the control laws make the system state converge to* $(0, 0)$ *from any the initial state* $(\theta_0, 0)$ *where* $\theta_H < \theta_0 < \theta_T$*.*

Note 1 *In the simulation, the control law does not always switch just on the bold line in Fig. 4, such as (29). Then, the vector field of the switching point points to the outward direction of (29) at the neighborhood of the origin. Therefore, the system does stay around the origin, but not converge exactly to it.*