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A circularly coupled oscillator system for relative phase regulation and its application to timing control of a multicylinder engine

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ABSTRACT

In this paper, distributed systems that consist of many locally connected subsystems, especially oscillators, and produce linear state relations, such as a state difference, are treated. The relations are defined between two connected subsystems, where their references are also assigned as a goal behavior simultaneously. The problem is: how subsystem dynamics are constructed to converge the relations to their references by only use of local operations, and how these references are adjusted if they are unachievable. To solve the above problems, a mathematical description of the subsystem interactions are clarified by extending a method based on the gradient dynamics. Then, the reference adjustment is defined so that the subsystem interactions decreases. As an example of this formulation, the relative phase control of the circularly coupled oscillator system is considered, where the oscillation with the uniform phase lag should be achieved. This oscillator system is applied to the timing controller for the multicylinder engine, and its effectiveness is discussed based on the simulation results.

1. Introduction

Spatiotemporal patterns are often observed in various fields [1]: convective rolls in a thermal liquid system [2], chemical oscillation in the BZ reaction [3], or neural activations or motions in biological systems [4–6]. These patterns are constructed from coordinated behavior of the many homogeneous components, such as molecules, neurons, or their ensembles. Although the effect of independent actions of the distributed elements is limited only to their neighborhood, such microscopic behavior produces a global pattern at the macroscopic level. This bottom–up approach allows the patterns to change with the situation. In the above examples, striped and hexagonal patterns emerge in convective patterns [2], or chemically impure substances determine whether a target or spiral pattern is produced [3]. The oscillatory behavior of a distributed system is occasionally described using a coupled oscillator system, where the pattern is represented by relative phases in the synchronization. Coupled oscillator systems are utilized to mathematically explain human motor behavior [7–9], quadruped locomotion [10,11], insects [12,13], swimming patterns [14,15], and even the actions of single cell amoeba [16]. They are effectively used in robots as a CPG (Central Pattern Generator) controller [17–19]. The mathematical analysis of coupled oscillator systems has been thoroughly discussed with numerical simulations [20–25]. This paper differs from these

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excellent studies in that the desired phase lags (i.e., reference relative phases) are initially explicitly assumed as the objective of the control and then the design of the oscillator interaction is considered to achieve this objective. Next, adaptive behavior is considered for adjusting the reference relative phases if they are inappropriate for the environment. This engineering viewpoint is import for providing more insight into the system design. As for the former problem, a method based on the gradient system was proposed [20]. In Section 2, this method is reviewed after formulating the problem with some assumptions, and then the mathematical formulation of the subsystem dynamics is discussed with focusing on the interactions. In Section 3, how is the situation in which the reference relations are unachievable is explained at first, and then an adjustment method of the unachievable reference is proposed based on the subsystem interactions. In Section 4, the relative phase regulation in coupled oscillator system is considered as an example, and is applied to the timing control of multicylinder engine. Finally, this paper is concluded in the Section 5.

2. Control in the distributed system

2.1. Problem setting

Yuasa and Ito proposed a method for controlling relative phases in a coupled oscillator system [20]. Their framework is based on parallel and distributed operations and is more general in the sense that linear relations between the one-dimensional state variables of two coupled subsystems can be regulated to their references. Of course, the linear relations include a difference in the state variables (the relative phase, in the case of a coupled oscillator system).

First, the following are assumed:

- The system under consideration consists of homogeneous subsystems.
- The state of the subsystem is represented by a scalar variable.
- The subsystems are connected locally, implying that there is no hub subsystem which all subsystems connect to.
- Two connected subsystems can exchange state variables, each affecting the state variable of the other. This is called "interaction".
- A constraint (i.e., a variable calculated from the states of the two coupled subsystems) is defined for each connection. This variable is called a "constraint variable".
- Each constraint variable possesses a reference value that represents a purpose of the entire system.

Under these assumptions, the problem is described as follows:

- Define the dynamics of each subsystem with local interaction such that the constraint variables are regulated to their references.

Here, dynamics with local interaction means, mathematically, that the system never contains any state variables other than those of the connected subsystems.

2.2. A design method based on a gradient system

The state of each subsystem is denoted by $q_i \in R(i = 1, \dots, M)$, where M is the number of the subsystems. Yuasa and Ito [20] formulate a case where the constraint variable p_k is given by the linear relation:

$$p_k = L_{ki}q_i - L_{kj}q_j \qquad (1)$$

This equation implies that the connection $k(k = 1, \dots, K)$ connects subsystem i and subsystem j, as shown in Fig. 1. This relation can be represented using the matrix form

$$P = LQ \qquad (2)$$

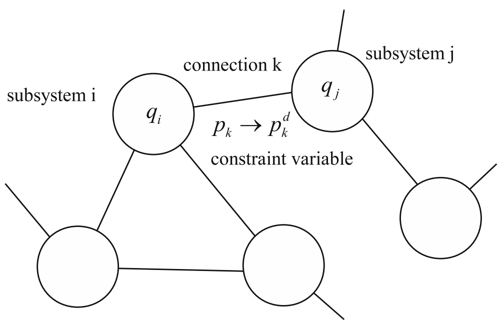


Fig. 1. System consisting of many homogeneous subsystem and constraint variable.

where $P = [p_1, \ldots, p_K]^T, Q = [q_1, \ldots, q_M]^T, L \in R^{K \times M}$ is a matrix whose (i,j) element is given as L_{ij}. Owing to (1), the number of non-zero elements is two in each row of the matrix L. Then, the problem is how to design subsystem dynamics

$$\dot{q}_i = f_i(Q_{N(i)}) \tag{3}$$

for P to converge to its reference $P^d = [p_1^d, \ldots, p_K^d]^T$. Here, $N(i)$ is a set of subsystems connected to subsystem i, $Q_{N(i)}$ is a vector in R^{N_i} whose element belongs to $N(i)$ and N_i is the number of the elements in $N(i)$.

A method using a gradient system has been proposed for this problem.

Theorem 1 [20]. *Suppose that the dynamics of the distributed system is given as*

$$\dot{Q} = \overline{F} + \widetilde{F} \tag{4}$$

Here, $\overline{F}$ is a vector within the kernel of matrix L and $\widetilde{F} = [\tilde{f}_1, \ldots, \tilde{f}_M]^T$ is described as a function of the variable x_i:

$$\tilde{f}_i = \tilde{f}_i(x_i) \tag{5}$$

where x_i is the ith element of the vector X that is defined as

$$X = -L^T P = -L^T L Q \tag{6}$$

Then, the dynamics of P becomes a gradient system whose potential function $V(P) = V_X(X)$ is given by

$$V_X(X) = \sum_{i=1}^{M} \int \tilde{f}_i(x_i) dx_i \tag{7}$$

Because of the definition of L, x_i contains no state variables other than those of subsystems connected to subsystem i, implying that the conditions in (3) are satisfied. $\tilde{f}_i(x_i)$ expresses the effect of interaction, because any other terms do not contain the states of connected subsystems.

2.3. Description of the effect from the connected subsystems

The above theorem results in the following subsystem dynamics:

$$\dot{q}_i = \bar{f}_i + \tilde{f}(x_i) \tag{8}$$

Now, what type of functional systems should be used to design $\tilde{f}(x_i)$?

The dynamics of the linear constraint variable P become

$$\dot{P} = L\widetilde{F} \tag{9}$$

If P is controlled to its reference P^d, the time evolution of P should stop, i.e., $\dot{P} = 0$ should hold. One method is to define $\widetilde{F}$ so that $\widetilde{F} = 0$ when $P = P^d$. When P becomes P^d, X is

$$X^d = -L^T P^d \tag{10}$$

From the above equation, the next relation is obtained:

$$X - X^d = -L^T(P - P^d) \tag{11}$$

Thus, it is sufficient to define $\tilde{f}$ as

$$\tilde{f}_i = g(x_i - x_i^d) \tag{12}$$

where the function g satisfies the following three conditions [26]:

$$g(0) = 0 \tag{13}$$

$$\left.\frac{dg(x)}{dx}\right|_{x=0} > 0 \tag{14}$$

$$g(x) \neq 0 (x \neq 0) \tag{15}$$

The first condition (13) ensures $\tilde{f} = 0$ at $x_i = x_i^d$. The next condition (14) is needed to make $x_i = x_i^d$ the minimum point of the potential function of (7). The last condition (15) is required for $x_i = x_i^d$ to be the sole minimum point of this potential function. The odd function should be selected if the effect of the connected subsystem is the same in the bilateral direction.

3. Constraint adjustment

3.1. Expression of interaction

In Section 2.1, the interaction was defined as an effect on the state of the other subsystem. Here, the expression of this interaction is discussed under the formulation in Section 2.3.

$\tilde{f}_i$ in (8) is defined to achieve the reference P^d. Therefore, one reasonable idea is that the effect from the other subsystem is driven by the error, i.e., the difference in the constraint variable p_k from its reference p_k^d. Among such expressions, the simplest one is linearly represented as

$$I_k = -K_k(p_k - p_k^d) \tag{16}$$

Thus, I_k is regarded as a mathematical expression of the interaction that works at connection k. K_k is a parameter that adjusts the magnitude of the interaction. If an appropriate value is set to the parameter K_k based on the matrix L, (11) is rewritten as

$$x_i - x_{di} = -\sum_{k=1}^{K}(L^T)_{ik}(p_k - p_k^d) = \sum_{k\in C(i)} I_k \tag{17}$$

where $C(i)$ is a set of connections that couple the subsystem i. The second equality holds because of the following reason: L_{ij}^T is not zero if there is a connection between subsystems i and j. In other words, it becomes zero only between unconnected subsystems, implying that the summation in (17) actually sums the interaction only from subsystems connected to subsystem i.

3.2. Unachievable reference

If the system contains a loop connection, such as in a circularly coupled system, the achievability of the reference is critical. For example, assume an oscillator system with n circularly coupled oscillators. A reference that causes each oscillator to oscillate with a $1/n$-period lag is certainly achievable, but a reference with $1/(n-1)$-period lag is not.

What happens in the stationary state if unachievable reference is set? The dynamics of P are given by (9), thus, $\dot{P} = 0$ gives its stationary state. Then, if $\widetilde{F}$, the orthogonal component of the kernel space of L, is defined using an odd function, as discussed in Section 2.3, $X - X^d = 0$ holds because $\widetilde{F} = 0$. Ideally, $X - X^d = 0$ should be equivalent to that P converges to P^d, i.e., $P - P^d = 0$. According to (11), however, $X - X^d = 0$ is not a sufficient but only a necessary condition for $P - P^d = 0$. In other words, if $P - P^d(\neq 0)$ stays at the kernel of L^T, $X - X^d$ becomes zero.

When $P - P^d \neq 0$ at the stationary state, the interaction in (16) does not disappear – it takes non-zero value. This implies that non-zero interactions work permanently. This situation is not desirable from two points of view. Interactions are supposed to work to achieve the references. However, in this situation, the references are never achieved, even if the interactions are always working. This is inconsistent from a viewpoint of system control. Secondarily, the stationary state is maintained by the mutual cancellation of non-zero interactions. Such non-zero interactions are disadvantageous from a cost viewpoint and occur when unachievable references are set. Thus, modification of the unachievable reference must be introduced.

3.3. Adjustment of unachievable reference

In this section, the adjustment of the unachievable reference is discussed. Taking the above disadvantages into consideration, reducing non-zero interactions is a natural criterion of the modification. Accordingly, using a cost function given as the squared sum of the interactions,

$$V_I = \frac{1}{2}\sum_{k=1}^{K} I_k^2 \tag{18}$$

the adjustment rule is defined to decrease it as

$$\frac{dp_i^d}{dt} = -\tau_p \frac{\partial V_I}{\partial p_i^d} \tag{19}$$

Here, τ_p is a parameter that regulates the adjustment speed of P^d. This is equivalent to memorizing a currently emerging P as P^d. Here, P^d obtained from the above rule is a reference appropriate to the current environmental conditions in the sense that it can be maintained with zero interactions. Note that this adjustment must be sufficiently slower than the dynamics (9) in order to produce an appropriate constraint variable P, because the adjustment should start after the dynamics of the constraint variable converges.

4. Application to timing control of multicylinder engine

4.1. Circularly coupled oscillator system

4.1.1. Formulation

The previous section proposed a distributed control method for controlling constraint variables between subsystems. Here, it is applied to the relative phase regulation of a circularly coupled oscillator system. In this system, N oscillators are connected in a circle, where each oscillator is numbered, in order from 1 to N, as shown in Fig. 2. The connections in this coupled oscillator system are local – oscillator i is connected only to neighboring oscillators, oscillators $i-1$ and $i+1$. The phase of each oscillator is $q_i (i = 1, \ldots, N)$.

The connection k represents a connection that connects oscillator k to oscillator $k+1$. To this connection k, a constraint variable p_k is defined. In this problem, the constraint variable is the difference of the states of the two connected oscillators, i.e., the relative phase. This is obviously a linear relation, given as

$$p_i = q_{i+1} - q_i \tag{20}$$

This matrix expression corresponding to (2) is given using the matrix L

$$L = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \ddots & \vdots \\ \vdots & 0 & -1 & \ddots & 0 \\ 0 & \vdots & \ddots & \ddots & 1 \\ 1 & 0 & \cdots & 0 & -1 \end{bmatrix} \tag{21}$$

The kernel of L, i.e., $\bar{f}$ becomes

$$\bar{f} = [1, \ldots, 1]^T \tag{22}$$

Regarding the orthogonal complementary space of this kernel, a sine function is selected as $g(x)$ in (12), taking the periodicity of the oscillator phase into account:

$$g(x) = \tau \sin(x) \tag{23}$$

Then, the oscillator dynamics are defined as follows:

$$\dot{q}_i = \omega + \tau \sin(q_{i-1} - 2q_i + q_{i+1} - p^d_{i-1} + p^d_i) \tag{24}$$

Here, ω is a constant corresponding to the natural angular frequency, $\tau > 0$ is a parameter adjusting the magnitude of the interactions, and $q_{N+1} = q_1, q_0 = q_N, p_{N+1} = p_1, p_0 = p_N, p^d_{N+1} = p^d_1, p^d_0 = p^d_N$.

If the potential function is defined using (7) as

$$V = \sum_{i=1}^{N} \tau \cos((p_{i-1} - p^d_{i-1}) - (p_i - p^d_i)) \tag{25}$$

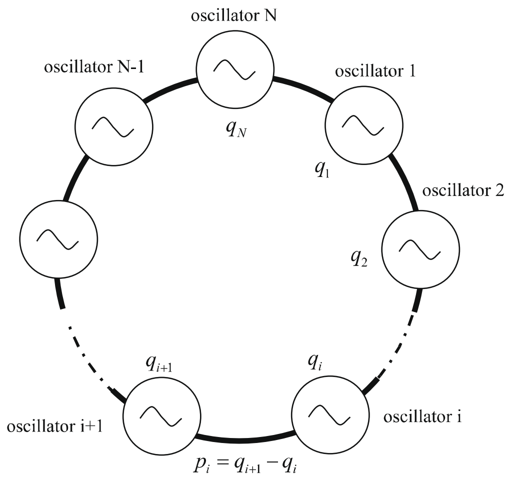


Fig. 2. A circularly coupled oscillator system.

The dynamics of p_i are certainly described as the gradient system of this potential function:

$$-\frac{\partial V}{\partial p_i} = \tau\sin((p_i - p_i^d) - (p_{i+1} - p_{i+1}^d)) - \tau\sin((p_{i-1} - p_{i-1}^d) - (p_i - p_i^d)) = \dot{q}_{i+1} - \dot{q}_i = \dot{p}_i \tag{26}$$

4.1.2. Stationary state

At the stationary state, $X - X^d = 0$ is satisfied. Using (11) and (21), the solutions of $X - X^d = 0$ are given as

$$p_1 = \cdots = p_K = const. \tag{27}$$

This means that an oscillation pattern finally emerges in which all oscillators oscillate with a constant relative phase to the connected one. Of course, this oscillation pattern includes a completely phase-coherent oscillation, where all the oscillators have the same phase, i.e., the zero relative phase.

4.1.3. Adjustment of the reference relative phase

In the previous section, an oscillation pattern with the same relative phases was achieved regardless of the reference P^d. Based on the analysis in Section 3, if the constraint variable p_k is not equal to the reference p_k^d in the stationary state, this reference is unachievable and inappropriate under the current conditions. In such a case, the reference is adjusted according to (19) to be appropriate and consistent with current conditions.

4.2. Application to the timing controller of a multicylinder engine

4.2.1. Concept

Timing control of a multicylinder engine is one possible example of the application of the circularly coupled oscillator system formulated in this section [27,28]. In a four-stroke cycle engine, four strokes are repeated: induction, compression, explosion, and exhaust. Because of its periodicity, this engine stroke is characterized by the phase of the engine cycle. Appropriate phase shifts among the cylinders provide stable output with minimal fluctuation. Inappropriate phase timings, however, causes vibration or degradations of engine efficiency. In short, 'homogeneous phase shifts' among the cylinders are important for a multicylinder engine. This requirement well matches the behavior of the circularly coupled oscillator system considered in Section 4.1.1.

The production of homogeneous phase shifts has no relationship with the reference of the constraint variable in the circularly coupled oscillator system. This may enhance the fault tolerance of this engine. For example, assume that one of the cylinders fails. In this situation, the relative phases of the engine cycle are not equal among the remaining cylinders. However, drive with the same interval is feasible with the remaining cylinders if the cylinder movement phase is adjusted properly. To this adjustment the above property must be applied. If possible, the relative oscillator phases should automatically change when the number of oscillators increases or decreases because of troubles or system modification.

In addition, if the reference is also adjusted based on the current oscillation, the coupled oscillator system can always memorize the appropriate oscillation pattern for the current condition.

4.2.2. Controller specifications

A four-stroke cycle engine is considered. The number of cylinders is set to N. The dynamics of this engine are described by the motion equation and the state equation for an ideal gas [29].

The specifications of the controller that regulates the timing of the engine cycle among the cylinders are set as follows:

- It produces an oscillation pattern with a $1/n$-period phase lag, if the number of cylinders in operation is n.
- It adjusts the relative phase automatically when the number of cylinders increases or decreases.
- It learns or memorizes the appropriate references for the current conditions.

Here, the controller is assumed to detect the rotational velocity of the engine shaft Ω.

4.2.3. Design and assumption

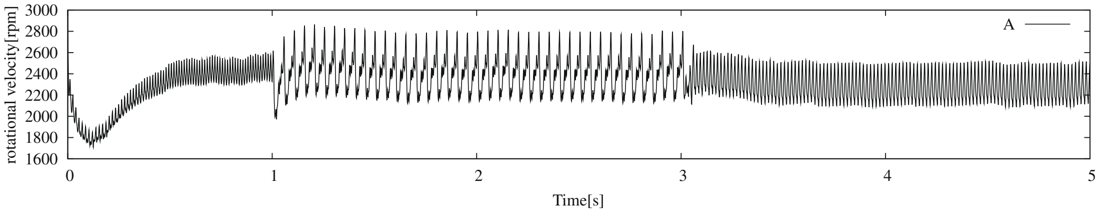
The circularly coupled oscillator system in Section 4.1 is used. N oscillators are prepared for the controller. This number is the same as that of the engine cylinder. An oscillator is assigned to each cylinder, and the phase of the engine cycle is assumed to be controlled by the phase of the corresponding oscillator.

The rotational velocity of the engine shaft Ω is sent from the engine to the controller as a feedback signal. Each oscillator in the controller is driven based on this signal, i.e., using the parameter ω in (24). This ω is determined from the rotational velocity of the shaft $\omega = \omega(\Omega)$ to synchronize the shaft and stroke cycles.

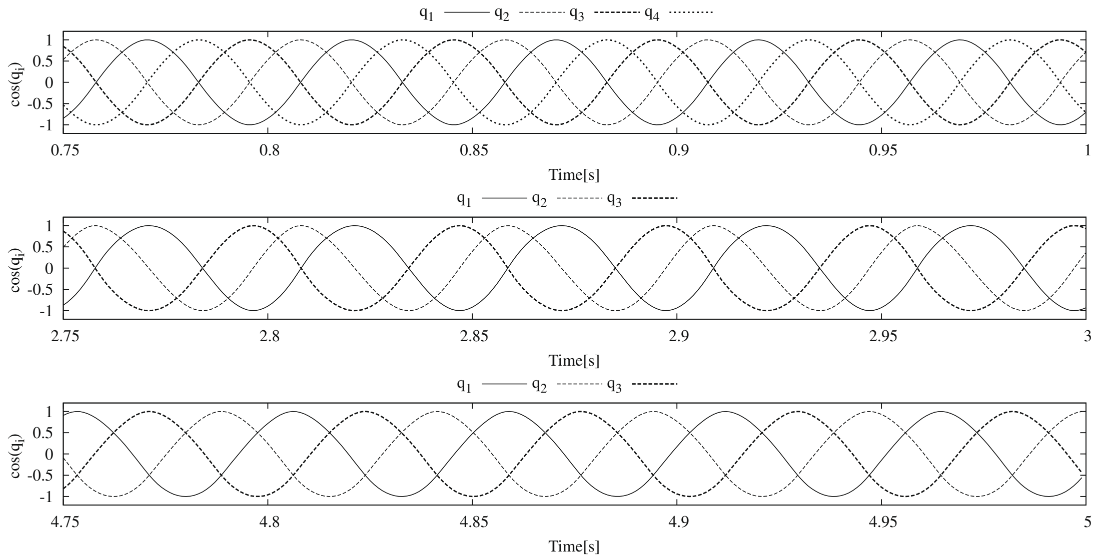
4.3. Simulations

4.3.1. Conditions

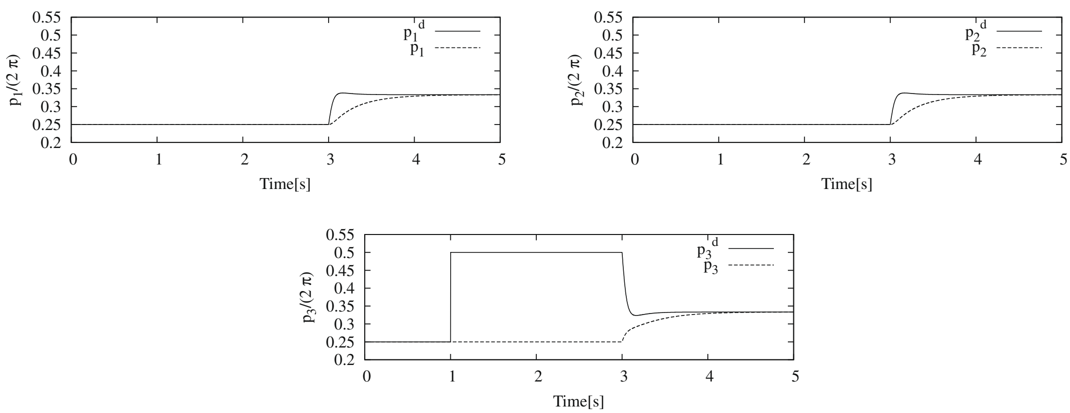
To examine the effect of homogeneous phase shifts, consider the case where the number of cylinders decreases/increases from initial four cylinders $(N = 4)$ as follows:



(a) Rotation velocity of the engine shaft.



(b) phase of oscillators.

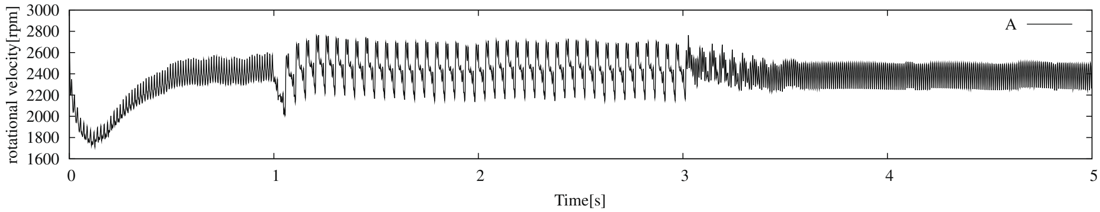


(c) relative phases and their references.

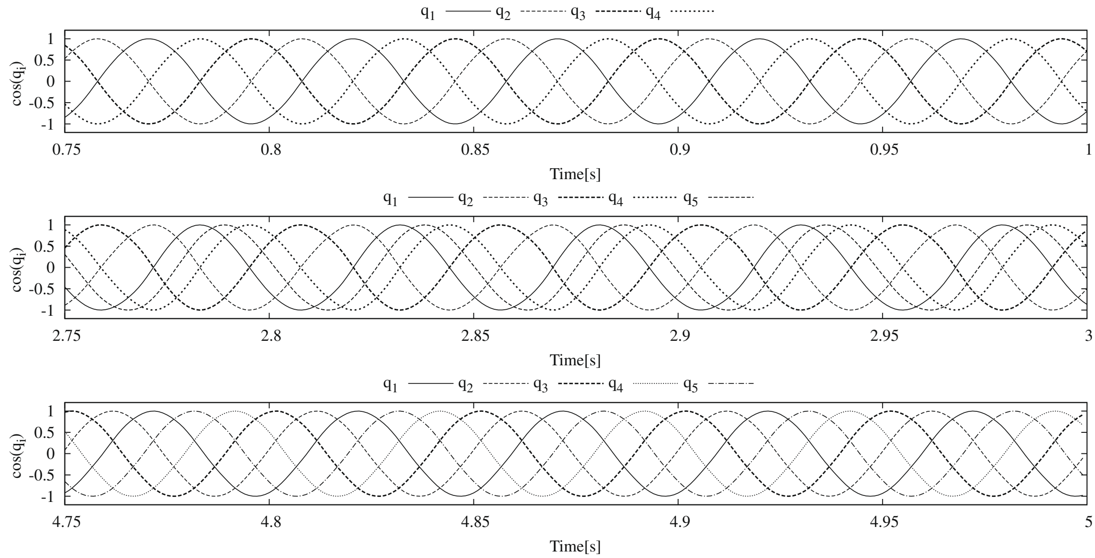
Fig. 3. Simulation results when the number of the operating cylinders decreases.

- The number of operating cylinder changes at 1.0 [s].
- For a while, the engine works without any phase shift.
- Oscillator control for relative phase regulation starts to operate at 3.0 [s].

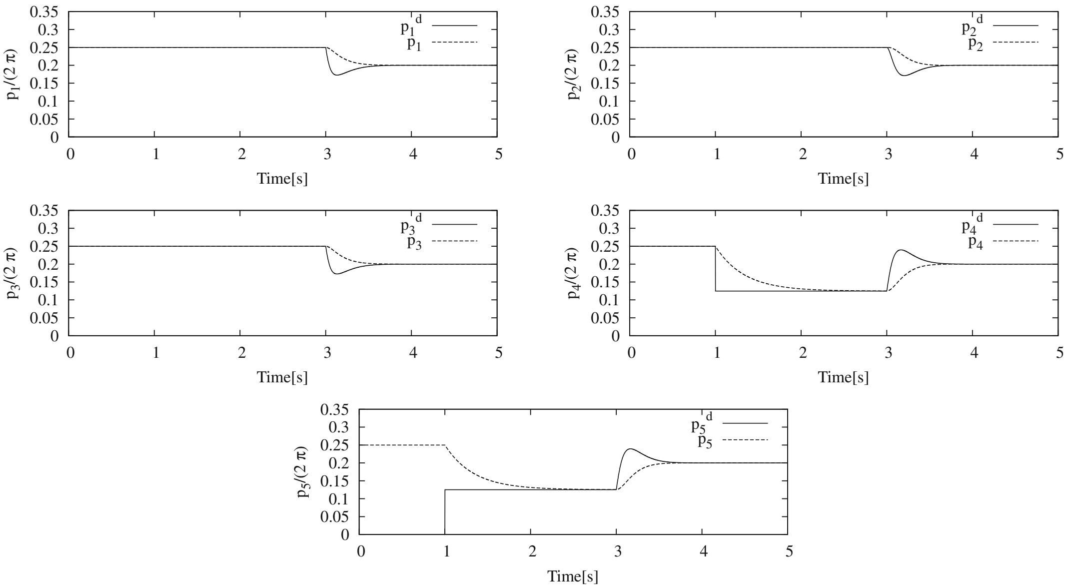
Here, it is assumed that the oscillator connection is kept in the circular form, regardless of the increment/decrement of the cylinders. Simulations are based on the following equations; (24) for oscillator dynamics, (19) for the adjustment of unachievable relative phase references, (31) for motion equations of engine with constraints (40) in appendix A, and gas dynamics defined separately in five phases in appendix B. The parameters are set as follows: $\tau = 10.0$ and $\tau_p = 0.5$. Because



(a) Rotation velocity of the engine shaft.



(b) phase of oscillators.



(c) relative phases and their references.

Fig. 4. Simulation results when the number of the operating cylinders increases.

the number of cylinders is known at the initial state ($N = 4$), each oscillator phase is set such that they are shifted $\pi/2$ from each other and the references of the relative phases satisfy this condition, i.e., $p_k^d = \pi/2 (k = 1, \ldots, 4)$. The oscillator is driven according to the feedback of the shaft speed followed by

Table 1
Numerical analysis of the shaft rotation velocity (rpm).

Condition	Average	S.D.	Maximum	Minimum	Fluctuation
4-Cylinders	2434.5	82.9	2601.3	2324.7	276.6
3-Cylinders (before control)	2404.9	168.4	2806.1	2143.6	662.5
3-Cylinders (under control)	2278.0	131.8	2278.0	2087.4	190.6
5-Cylinders (before control)	2431.5	147.6	2679.2	2146.4	532.8
5-Cylinders (under control)	2390.1	88.9	2501.6	2246.8	254.9

$$\omega = \Omega/2 (= \dot{\theta}/2) \tag{28}$$

because the piston moves up and down twice in the one engine cycle. The fourth order Runge–Kutta method are used for simulations with a step size of 0.1 [ms].

4.3.2. Results

Two cases are examined by the computer simulations. In the fist case, the number of cylinders decreases, i.e., cylinder four breaks down. The results are shown in Fig. 3. In the second case, the cylinder 5 is inserted between cylinder 4 and cylinder 1, resulting that the number of cylinders increases. The results are shown in Fig. 4. The graphs in (a)–(c) are time-based plots of the rotational velocity of the engine shaft Ω, the phase of each oscillator, and the relative phases with their references, respectively. The shaft rotational velocity at each condition is summarized in Table 1. In both cases, the notation "4 cylinders" denotes the results during the period from 0.9 s to 1.0 s. And, the notation "before control" means the results during the period from 2.9 s to 3.0 s, while "under control", from 4.9 s to 5.0 s.

4.3.3. Discussions

The shaft rotational velocity settled into the stationary state after a transient period of about 0.5 s, as shown in the graph (a) of both Figs. 3 and 4. The variation of the number of operating cylinders noticeably enlarges the fluctuation of the shaft rotation velocity, which is observed as a phase transition at 1.0 s. However, this fluctuation is somewhat suppressed thanks to the relative phase regulation of the oscillator control. Numerical results are summarized in Table 1.

Initially, each oscillator oscillates with the same phase lags, i.e., $\pi/2$, as depicted in the top graph of Fig. 3(b) and Fig. 4(b). This balanced oscillation pattern is disturbed as shown in the middle graph. The relative phase control, however, recovers the homogeneous phase shift as shown in the bottom graph: the relative phase and its desired values converge to $2\pi/3$ (1/3-period) in Fig. 3(c) while to $2\pi/5$ (1/5-period) in Fig. 4(c), which is an appropriate and achievable relative phase in both cases.

Regarding to the convergence of the relative phase control, the ideal object $X - X_d = 0$ is difficult to achieve due to the parameter variations among cylinders, or the distortion of the sinusoidal oscillation caused by the intermittent driving force from each cylinder. Therefore, taking the periodicity of the oscillator dynamics into account, a criterion $\int_T (X - X_d) dt < \varepsilon$ (here T is a period of the oscillation, and ε is a threshold that determines a convergence) is more appropriate, although the convergence was not mainly considered in the simulations. This criterion will somewhat loosen the homogeneous assumption of subsystems in Section 2.1.

In order to realize this system as the actual mechanical engine, we have to make an effort to remove two assumptions: one assumption is that the phase of cylinder movement can be adjusted according to the oscillator signal, which is normally determined mechanically as a relative angle of the clank shaft. The other is how to keep the oscillator connection as a circular form.

5. Concluding remarks

This paper addressed a problem in a distributed system that the linear state relations between subsystems are regulated to its reference by means of a local parallel control. Here, the local means that the controller utilizes restricted information given only from the connected subsystems. Based a method using a gradient system, it was clarified that monotonically increasing odd functions are available to describe the effect from the connected subsystems. Next, the adjustment of the references was discussed when the references were unachievable. Making the definition of the subsystem interactions clear, an rule of references adjustment was proposed so as to reduce these interactions. As an example of the distributed system, a relative phase regulation of the circularly coupled oscillator system was considered. Then, it was applied to the timing control of the multicylinder engine. Computer simulations demonstrated the effective change of the linear state relations, i.e., the relative phase, to decrease the fluctuation of the rotation velocity of the engine shaft even if the number of operating cylinder varied. In addition, the reference of the relative phase was adjusted to an appropriate and achievable one in the current conditions. As a future works, we try to extend this control and adjustment method so that it can treat non-linear state relations and consider several practical applications of this method.

Appendix A. Motion equation of the single cylinder with constraints

The motion is described within the two-dimensional plane. At first, we describe the mechanical constraints. Notations are shown in Fig. 5. From the positional relation, the following equations are satisfied:

$$\boldsymbol{C}(\boldsymbol{X}) = \begin{bmatrix} X_p - X - \ell\cos\phi \\ -Y + \ell\sin\phi \\ X - \ell\cos\phi - r\cos\theta \\ Y + \ell\sin\phi - r\sin\theta \end{bmatrix} = \boldsymbol{0} \tag{29}$$

where

$$\boldsymbol{X} = [X_p \quad X \quad Y \quad \phi \quad \theta]^T \tag{30}$$

is a position vector of the motion equation. Then, the motion equation under these constraints are given as follows:

$$\boldsymbol{M}\ddot{\boldsymbol{X}} = \boldsymbol{J}^T\boldsymbol{F} + \boldsymbol{F_e} \tag{31}$$

Here

$$\boldsymbol{M} = diag[M_p, M, M, I, I_s] \tag{32}$$

M, M_p are mass of the con rod and piston and I, I_c are the inertial moment of the con rod and shaft, respectively.

$$\boldsymbol{J} = \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{X}} = \begin{bmatrix} 1 & -1 & 0 & \ell s_\phi & 0 \\ 0 & 0 & -1 & \ell c_\phi & 0 \\ 0 & 1 & 0 & \ell s_\phi & rs \\ 0 & 0 & 1 & \ell c_\phi & -rc \end{bmatrix} \tag{33}$$

where $s_\phi = \sin\phi, c_\phi = \cos\phi, s = \sin\theta$ and $c = \cos\theta$

$$\boldsymbol{F} = [F_{px} \quad F_{py} \quad F_{cx} \quad F_{cy}]^T \tag{34}$$

is constraint force exerted between links.

$$\boldsymbol{F}_e = [-F_A + F_f - M_p g \quad -Mg \quad 0 \quad \tau_p - \tau_c \quad \tau_c]^T \tag{35}$$

F_A is force generated by the pressure difference:

$$F_A = (P - P_0)S \tag{36}$$

S is the area of the cylinder. F_f, τ_p and τ_c are the friction torque/force given as

$$\tau_c = -B_c(\dot{\phi} + \dot{\theta}) \tag{37}$$

$$\tau_p = -B_p\dot{\phi} \tag{38}$$

$$F_f = -B_f\dot{X}_p - \mu_f F_{py} sgn(\dot{X}_p) \tag{39}$$

B_c, B_p and B_f denotes the viscosity and μ_f is the kinetic friction coefficient of the cylinder.

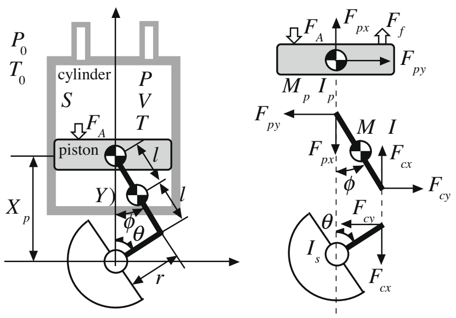


Fig. 5. Mechanical model of engine.

Differentiating the constraints (29) two times, we obtain

$$\boldsymbol{J}\ddot{\boldsymbol{X}} = \dot{\boldsymbol{J}}\dot{\boldsymbol{X}} \tag{40}$$

The simultaneous Eqs. (31) and (40) have the unique solution $\ddot{\boldsymbol{X}}$ and $\boldsymbol{F}$ since matrix M is non-singular. This numerical solution is utilized in the computer simulations.

Of course, in the case of the multicylinder system, the motion equations of each piston and con rod should be added. In addition, that of the shaft must be modified so that the driving moment is applied from the all the cylinder.

Appendix B. Gas dynamics

The gas dynamics are expressed in each stroke. The stroke is assumed to be switched based on the crank shaft angle. However, the volume is uniquely demerited from the position of the piston X_p at all the strokes.

$$V = S(X_M - X_p) + V_0 \tag{41}$$

where X_M is the position of the piston at the top dead center, V_0 is the volume of the initial state. From this equation, the dynamics of the volume is defined as

$$\dot{V} = -S\dot{X}_p \tag{42}$$

B.1. Induction stroke ($0 \leqslant \theta \leqslant \pi/2$)

This is a stroke from top dead center ($\theta = 0$) to the bottom dead center ($\theta = \pi/2$). The temperature T_1 as well as the pressure P_1 is kept at the initial value, i.e., $P_1 = P_0$ and $T_1 = T_0$, where P_0 and T_0 are constants denoting pressure and temperature of the outside air, respectively. Thus, the dynamics of T_1 and P_1 is given as

$$\dot{P}_1 = 0 \tag{43}$$

$$\dot{T}_1 = 0 \tag{44}$$

B.2. Compression stroke ($\pi/2 \leqslant \theta \leqslant \pi$)

This stroke is assumed to end when the piston reaches the top dead center again, i.e., $\theta = \pi$. In this stroke, the pressure and temperature is denoted by P_2 and P_2. If the adiabatic assumption is applied, we obtain

$$P_2 V_2^{\kappa} = P_1^e (V_1^e)^{\kappa} \tag{45}$$

Here, P_1^e and V_1^e are a pressure and temperature at the end of the induction stroke, and κ is the ratio of specific heat. In addition, from the state equation of the gas, we get

$$\frac{P_2 V_2}{T_2} = \frac{P_1^e V_1^e}{T_1^e} \tag{46}$$

Using above two equations, the following equations hold:

$$P_2 = P_1^e \cdot \epsilon_2^{\kappa} \tag{47}$$

$$T_2 = T_1^e \cdot \epsilon_2^{\kappa-1} \tag{48}$$

where

$$\epsilon_2 = \frac{V_1^e}{V_2} \tag{49}$$

Then, the dynamics of P_2 and P_3 are given as

$$\dot{P}_2 = P_1^e \cdot \kappa \epsilon_2^{\kappa-1} \cdot \dot{\epsilon}_2 \tag{50}$$

$$\dot{T}_2 = T_1^e \cdot (\kappa - 1)\epsilon_2^{\kappa-2} \cdot \dot{\epsilon}_2 \tag{51}$$

$$\dot{\epsilon}_2 = -V_1^e \frac{\dot{V}_2}{V_2^2} \tag{52}$$

B.3. Explosion stroke ($\pi \leqslant \theta \leqslant 3\pi/2$)

The explosion stroke is considered by dividing it into two strokes: combustion stroke and expansion stroke.

B.3.1. Combustion process ($\pi \leqslant \theta \leqslant \pi + \theta_C$)

The combustion process starts exactly at the phase $\theta = \pi$ and ends at $\theta = \pi + \theta_C$. There, the heat is assumed to be provided in proportion to the deviation of the phase

$$\Delta T_3 = Q \cdot \Delta\theta \tag{53}$$

which allows the total amount of the heat to be kept regardless of the rotation speed. Here, T_3 is the temperature in the combustion process. Furthermore, the Boyle-Charle's law holds in this process

$$\frac{P_3 V_3}{T_3} = \frac{P_2^e V_2^e}{T_2^e} \tag{54}$$

where P_3 and V_3 are the pressure and volume in this process, and P_2^e, V_2^e and T_2^e are the pressure, volume and temperature at the end of the compression stroke. From them, the following evolution equations are obtained.

$$\dot{T}_3 = Q \cdot \dot{\theta} \tag{55}$$

$$\dot{P}_3 = P_2^e \left[\frac{\dot{T}_3}{T_2^e} - \frac{\dot{V}_3}{V_2^e} \right] \tag{56}$$

B.3.2. Expansion process ($\theta_c \leqslant \theta \leqslant 3\pi/2$)

As is the same as the compression stroke, the adiabatic assumption is imposed. The next equations are obtained in the similar way.

$$\dot{P}_4 = P_3^e \cdot \kappa \epsilon_4^{\kappa-1} \cdot \dot{\epsilon}_4 \tag{57}$$

$$\dot{T}_4 = T_3^e \cdot (\kappa - 1) \epsilon_4^{\kappa-2} \cdot \dot{\epsilon}_4 \tag{58}$$

$$\dot{\epsilon}_4 = -V_3^e \frac{\dot{V}_4}{V_4^2} \tag{59}$$

where P_4, V_4 and T_4 are the pressure, volume and temperature in this process, and P_3^e, V_3^e and T_3^e are the pressure, volume and temperature at the end of the combustion process.

B.4. Exhaust stroke ($3\pi/2 \leqslant \theta \leqslant 2\pi$)

The piston moves from the bottom ($\theta = 3\pi/2$) to top dead center ($\theta = 2\pi$). The pressure and temperature is assumed to be kept constant

$$P_5 = P_0 \tag{60}$$

$$T_5 = T_4^e \tag{61}$$

where P_5 and T_5 are the pressure and temperature in this stroke, and T_4^e is the temperature at the end of the explosion stroke. Thus, we get

$$\dot{P}_5 = 0 \tag{62}$$

$$\dot{T}_5 = 0 \tag{63}$$

Appendix C. Parameters in simulation

The following values were used in computer simulations. $M_p = 3.5$ kg,$M = 0.6767$ kg,$I = 0.005$ kgm,$I_c = 0.1$ kgm,$S = 0.007854$ m^2, $\ell = 0.075$ m,$r = 0.06$ m,$B_p = $ 5.0 Ns/m,$B_c = 0$ Ns/m,$B_f = 0$ Ns/m,$\mu_f = 0.3$, $P_0 = 103334$ Pa,$T_0 = 298$ K,$V_0 = 0.000058905$ m^3, $\theta_C = 0.3$ rad,$\kappa = 1.4$. Not that it was assumed that all cylinders are completely same. The Q is determined based on the shaft rotation velocity so that it goes to 2500 rpm.

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